MAT 142: Composition of Functions

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June 2, 2026

Reminders

At our last meeting, we discussed each of the following:

- The limit of a function as a generalization of function evaluation.
- Reading limits from a graph, including one-sided limits.
- Evaluating limits algebraically via direct substitution and the factor-and-reduce technique, when possible.
- The slope of the secant line as an average rate of change.
- The derivative as the limit of the difference quotient.

Try the following warm-up problems.

Problem 1: Consider $f(x) = 8 - x^2$. Find $f(0)$, $f(5)$, $f(-2)$, $f(x+h)$, and $f\left(x^2 - 5\right)$.

Problem 2: Consider $g(x) = \dfrac{x^2 + x}{x + 1}$. Find $g(0)$, $g(5)$, $g(-2)$, $g(x+h)$, and $g(2 - x)$.

Motivating Application

A forest fire leaves behind an area of burned grass in an expanding circular pattern. The radius of the circle is increasing with time according to

$$r(t) = 2t + 1$$

where t is in minutes and r is in feet.

Express the **area burned** as a function of **time** t. Find the total area burned after 10 minutes.

Before we work through this, consider the following:

- What is the formula for the burned area in terms of the radius?
- Is the radius truly the *independent* variable for the area function?
- What is the radius of the burn after 5 minutes? What is the area burned at that point?

We'll return to this application at the end of today's class.

Objectives

Today we introduce a powerful new way to combine functions.

After today's class meeting, you should be able to:

- Understand what it means to compose two functions.
- Construct the algebraic definition of a composed function $(f \circ g)(x)$.
- Evaluate a composed function at a given input value.
- Read composed function values from graphs.
- Recognize when composition arises naturally in applied settings.

Composition of Functions

Definition (Composition of Functions): Given two functions $f(x)$ and $g(x)$, as long as the range of g is a subset of the domain of f, we define the **composition** of f and g as:

$$(f \circ g)(x) = f(g(x))$$

The input x is first transformed by g, and then the output of g becomes the input for f.

Think of it as a two-stage machine:

$$x \longrightarrow \boxed{\;g\;} \longrightarrow g(x) \longrightarrow \boxed{\;f\;} \longrightarrow f(g(x))$$

Order of Composition Matters

Order matters. $(f \circ g)(x)$ and $(g \circ f)(x)$ are generally *not* equal – applying g first then f is different from applying f first then g.

Completed Example 1

Problem: Consider $f(x) = x + 4$ and $g(x) = x^2$.

(a) Determine $(f \circ g)(-7)$ by first computing $g(-7)$, then using the result as the input to f.

(b) Find the algebraic definition of $(f \circ g)(x)$ by simplifying $f(g(x))$, then evaluate $(f \circ g)(-7)$ directly.

Solution to (a). First, evaluate g at -7:

$$g(-7) = (-7)^2 = 49$$

Now use 49 as the input to f:

$$f(49) = 49 + 4 = \boxed{53}$$

Completed Example 1 (Cont'd)

Problem: Consider $f(x) = x + 4$ and $g(x) = x^2$. Find $(f \circ g)(x)$ algebraically, then evaluate at $x = -7$.

Solution to (b). Substitute $g(x)$ in place of every x in $f(x)$:

$$(f \circ g)(x) = f(g(x))$$

Completed Example 1 (Cont'd)

Problem: Consider $f(x) = x + 4$ and $g(x) = x^2$. Find $(f \circ g)(x)$ algebraically, then evaluate at $x = -7$.

Solution to (b). Substitute $g(x)$ in place of every x in $f(x)$:

$$\begin{aligned}(f \circ g)(x) &= f(g(x)) \\ &= f\left(x^2\right)\end{aligned}$$

Completed Example 1 (Cont'd)

Problem: Consider $f(x) = x + 4$ and $g(x) = x^2$. Find $(f \circ g)(x)$ algebraically, then evaluate at $x = -7$.

Solution to (b). Substitute $g(x)$ in place of every x in $f(x)$:

$$\begin{aligned}(f \circ g)(x) &= f(g(x)) \\ &= f\left(x^2\right) \\ &= x^2 + 4\end{aligned}$$

Completed Example 1 (Cont'd)

Problem: Consider $f(x) = x + 4$ and $g(x) = x^2$. Find $(f \circ g)(x)$ algebraically, then evaluate at $x = -7$.

Solution to (b). Substitute $g(x)$ in place of every x in $f(x)$:

$$\begin{aligned} (f \circ g)(x) &= f(g(x)) \\ &= f\left(x^2\right) \\ &= x^2 + 4 \end{aligned}$$

Now evaluate at $x = -7$:

$$(f \circ g)(-7) = (-7)^2 + 4 = 49 + 4 = \boxed{53}$$

Both methods give the same result – this will always be the case.

Composition of Functions Practice I

Try It! 1: Using the same functions $f(x) = x + 4$ and $g(x) = x^2$, evaluate $(g \circ f)(-7)$ using both methods.

What do you notice? What does this tell us about the order in which we compose functions?

Reminder for method 1: Start by evaluating $f(-7)$, then use that result as the input to g.

Reminder for method 2: Find $(g \circ f)(x) = g(f(x))$ algebraically first, then plug in -7.

Completed Example 2

Problem: Consider $f(x) = \dfrac{x}{x-5}$ and $g(x) = x^2 - 5$.

Construct $(f \circ g)(x)$ and evaluate it at $x = 0$, $x = 5$, and $x = \sqrt{5}$.

Solution. We substitute $g(x)$ into f:

$$(f \circ g)(x) = f(g(x))$$

Completed Example 2

Problem: Consider $f(x) = \dfrac{x}{x-5}$ and $g(x) = x^2 - 5$.

Construct $(f \circ g)(x)$ and evaluate it at $x = 0$, $x = 5$, and $x = \sqrt{5}$.

Solution. We substitute $g(x)$ into f:

$$\begin{aligned} (f \circ g)(x) &= f(g(x)) \\ &= f\left(x^2 - 5\right) \end{aligned}$$

Completed Example 2

Problem: Consider $f\left(x\right) = \dfrac{x}{x-5}$ and $g\left(x\right) = x^2 - 5$.

Construct $\left(f \circ g\right)\left(x\right)$ and evaluate it at $x = 0$, $x = 5$, and $x = \sqrt{5}$.

Solution. We substitute $g\left(x\right)$ into f:

$$\begin{aligned}
\left(f \circ g\right)\left(x\right) &= f\left(g\left(x\right)\right) \\
&= f\left(x^2 - 5\right) \\
&= \frac{x^2 - 5}{\left(x^2 - 5\right) - 5}
\end{aligned}$$

Completed Example 2

Problem: Consider $f\left(x\right) = \dfrac{x}{x-5}$ and $g\left(x\right) = x^2 - 5$.

Construct $\left(f \circ g\right)\left(x\right)$ and evaluate it at $x = 0$, $x = 5$, and $x = \sqrt{5}$.

Solution. We substitute $g\left(x\right)$ into f:

$$\begin{aligned}
\left(f \circ g\right)\left(x\right) &= f\left(g\left(x\right)\right) \\
&= f\left(x^2 - 5\right) \\
&= \frac{x^2 - 5}{\left(x^2 - 5\right) - 5} \\
&= \frac{x^2 - 5}{x^2 - 10}
\end{aligned}$$

Completed Example 2

Problem: Consider $f(x) = \dfrac{x}{x-5}$ and $g(x) = x^2 - 5$.

Construct $(f \circ g)(x)$ and evaluate it at $x = 0$, $x = 5$, and $x = \sqrt{5}$.

Solution.

$$(f \circ g)(x) = \frac{x^2 - 5}{x^2 - 10}$$

Now evaluate at each input:

$$(f \circ g)(0) = \frac{0-5}{0-10} = \frac{-5}{-10} = \boxed{\frac{1}{2}}$$

$$(f \circ g)(5) = \frac{25-5}{25-10} = \frac{20}{15} = \boxed{\frac{4}{3}}$$

$$(f \circ g)\left(\sqrt{5}\right) = \frac{5-5}{5-10} = \frac{0}{-5} = \boxed{0}$$

Composition of Functions Practice II

Try It! 2: Consider the functions $f(x) = 3x + 8$, $g(x) = \dfrac{x-8}{3}$, and $h(x) = x^2 - 5$.

- Construct $(f \circ g)(x)$, and use it to evaluate:
  - $(f \circ g)(0)$
  - $(f \circ g)(-8)$
  - $(f \circ g)\left(\sqrt{2}\right)$
- Construct $(g \circ f)(x)$, and use it to evaluate:
  - $(g \circ f)(-12)$
  - $(g \circ f)\left(\frac{3}{4}\right)$
  - $(g \circ f)(\pi)$
- What does this tell you about the functions f and g?
- Construct $(f \circ h)(x)$, and use it to evaluate:
  - $(f \circ h)(0)$
  - $(f \circ h)(-8)$
  - $(f \circ h)\left(\sqrt{2}\right)$
- Construct $(h \circ f)(x)$, and use it to evaluate:
  - $(h \circ f)(-12)$
  - $(h \circ f)\left(\frac{3}{4}\right)$
  - $(h \circ f)(\pi)$
- Construct $(g \circ g)(x)$, and use it to evaluate:
  - $(g \circ g)(0)$
  - $(g \circ g)(5)$
  - $(g \circ g)\left(\sqrt{5}\right)$

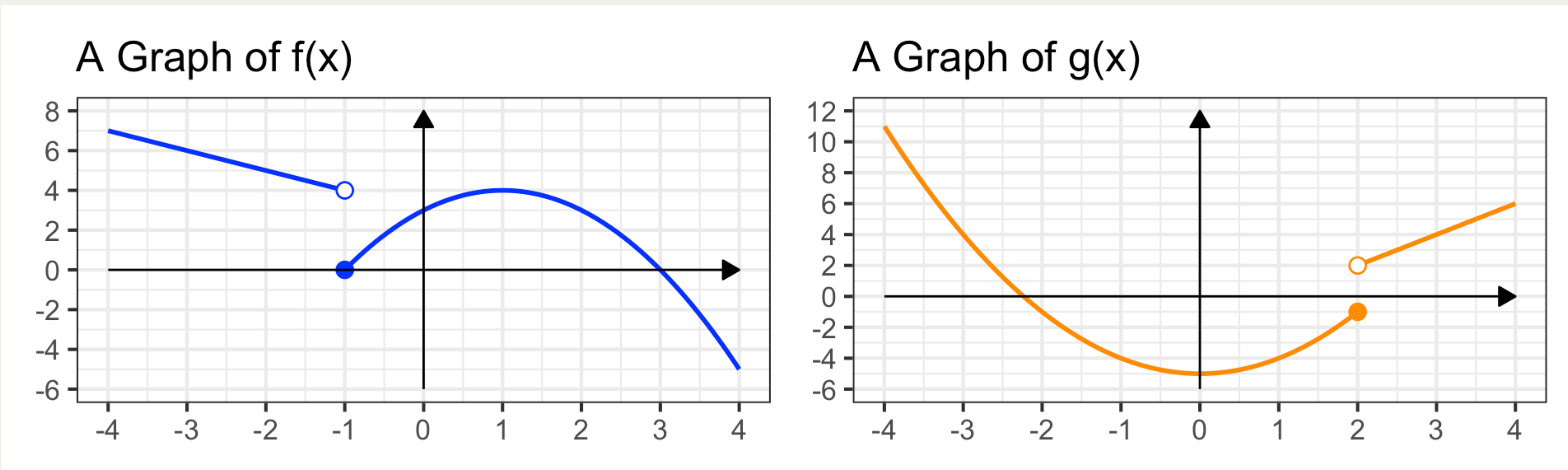
Composition from Graphs

We can also evaluate compositions from graphs. The key is working from the **inside out**: evaluate the inner function first, then use the result as the input to the outer function.

Problem: The graphs of $f(x)$ and $g(x)$ appear below. Use them to evaluate each of the following.

$$(a)\ \ (f \circ g)(2) \qquad (b)\ \ f(g(-1)) \qquad (c)\ \ (g \circ f)(0) \qquad (d)\ \ (g \circ f)(-1)$$

$$(e)\ \ (g \circ g)(-3) \qquad (f)\ \ (f \circ f)(-1) \qquad (g)\ \ f(g(f(2))) \qquad (h)\ \ g(f(g(2)))$$



Solution. Working from the inside out in each case:

- $(f \circ g)(2) = f(g(2)) = f(-1) = \boxed{0}$ *(closed dot at $x = -1$ on f)*

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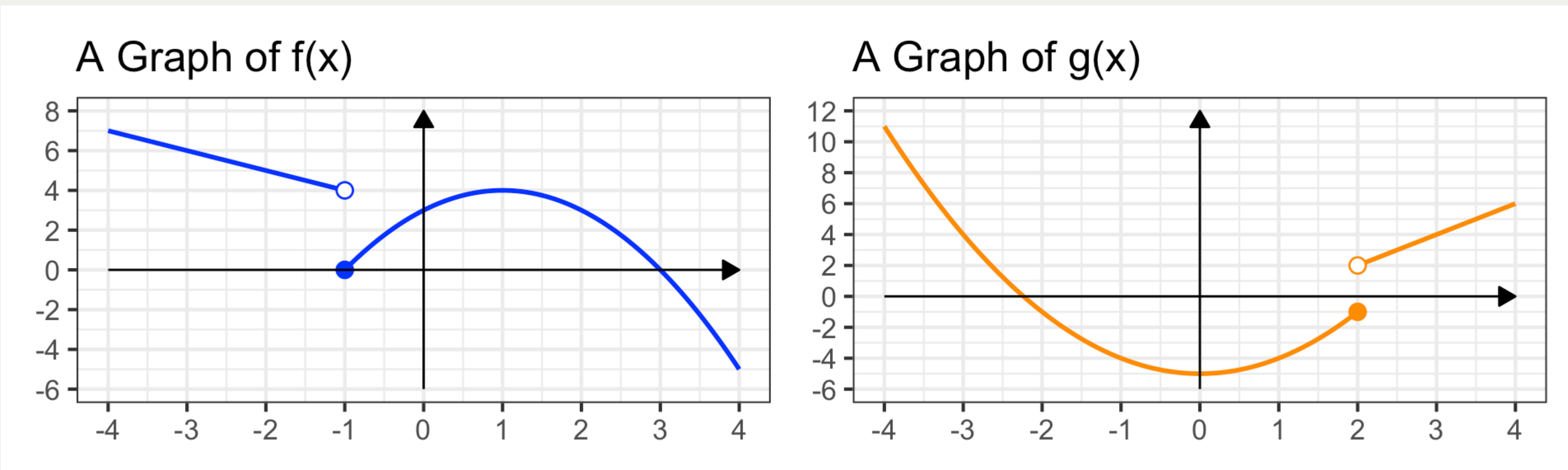

Solution. Working from the inside out in each case:

- $f(g(-1)) = f(-4) = \boxed{7}$

Composition from Graphs

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Problem: The graphs of $f(x)$ and $g(x)$ appear below. Use them to evaluate each of the following.

$$(a)\;\; (f \circ g)(2) \qquad (b)\;\; f(g(-1)) \qquad (c)\;\; (g \circ f)(0) \qquad (d)\;\; (g \circ f)(-1)$$

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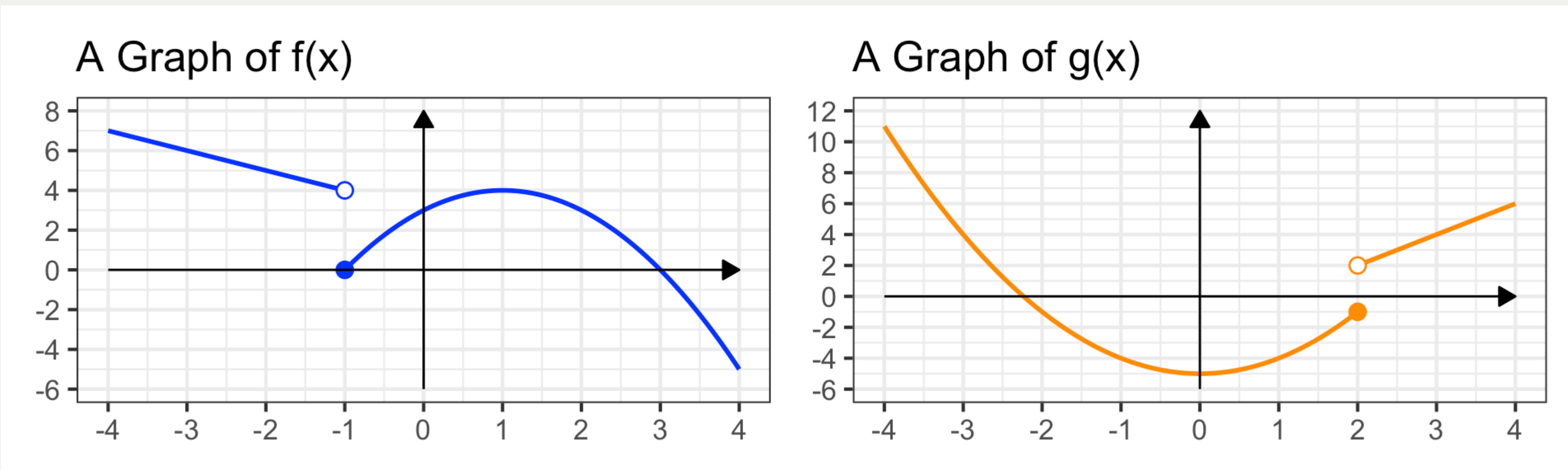

Solution. Working from the inside out in each case:

- $(g \circ f)(0) = g(f(0)) = g(3) = \boxed{4}$

Composition from Graphs

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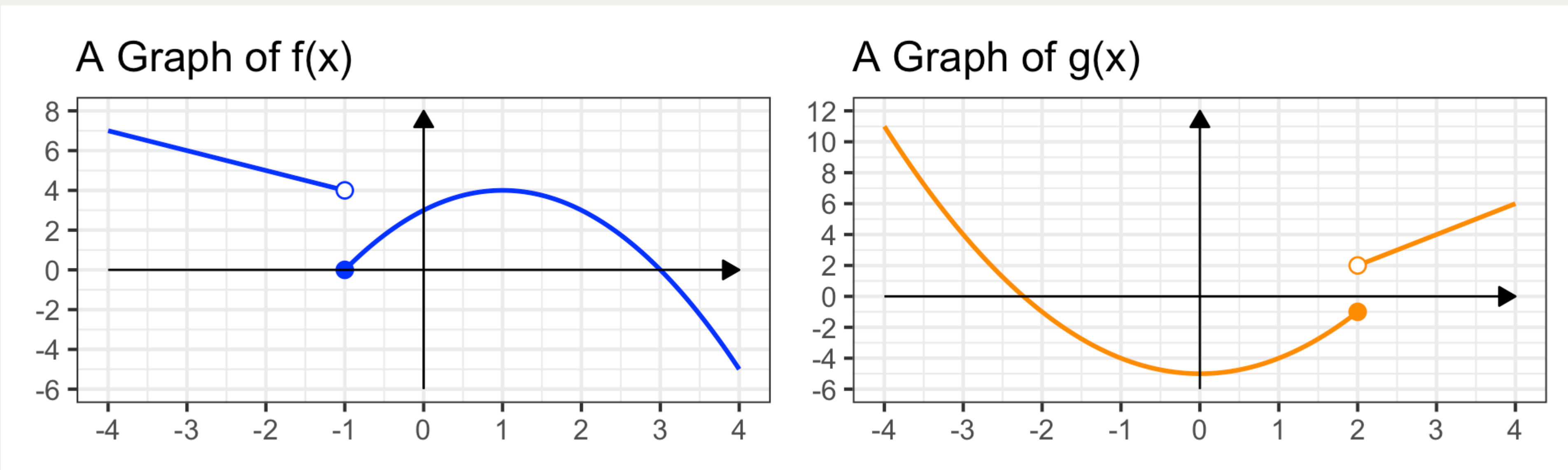

Solution. Working from the inside out in each case:

- $(g \circ f)(-1) = g(f(-1)) = g(0) = \boxed{-5}$

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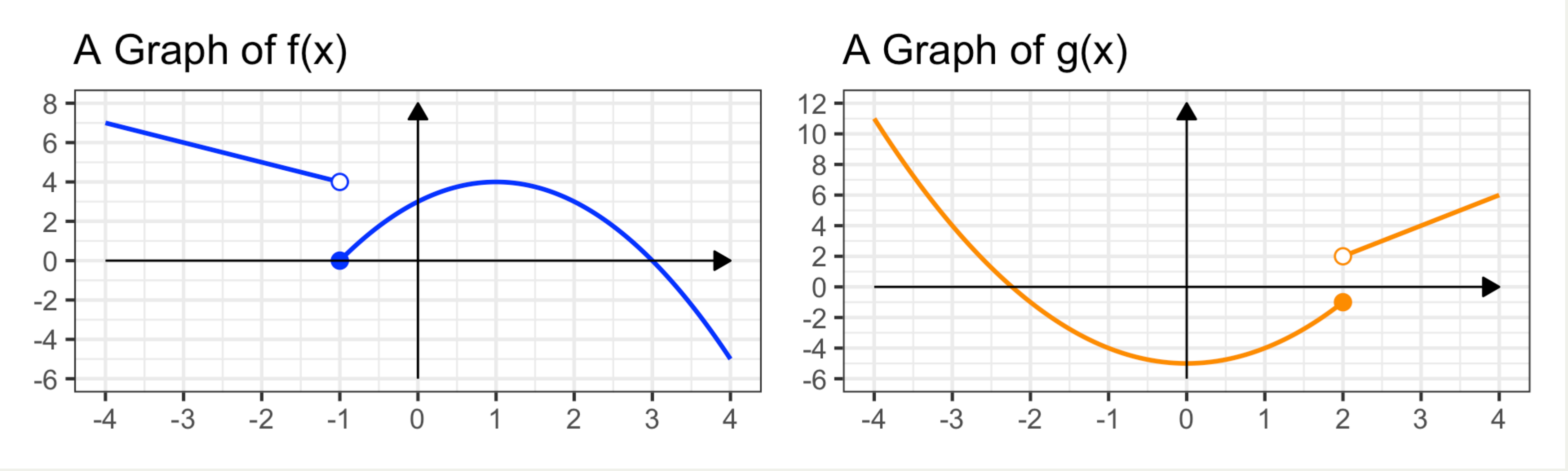

Solution. Working from the inside out in each case:

- $(g \circ g)(-3) = g(g(-3)) = g(4) = \boxed{6}$

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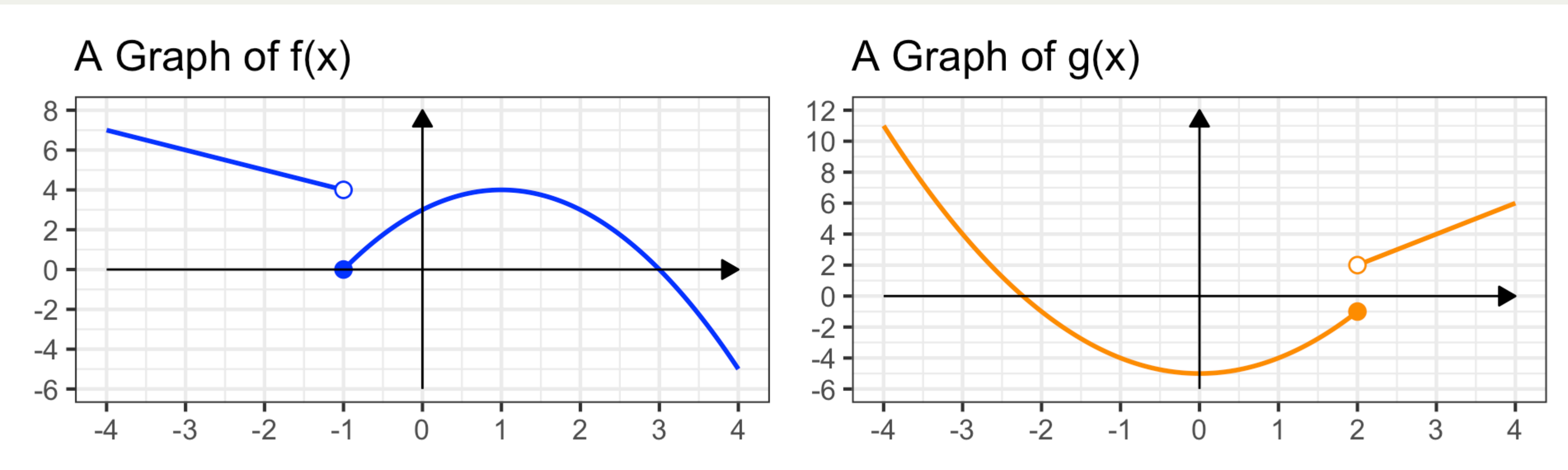

Solution. Working from the inside out in each case:

- $(f \circ f)(-1) = f(f(-1)) = f(0) = \boxed{3}$

Composition from Graphs

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Problem: The graphs of $f(x)$ and $g(x)$ appear below. Use them to evaluate each of the following.

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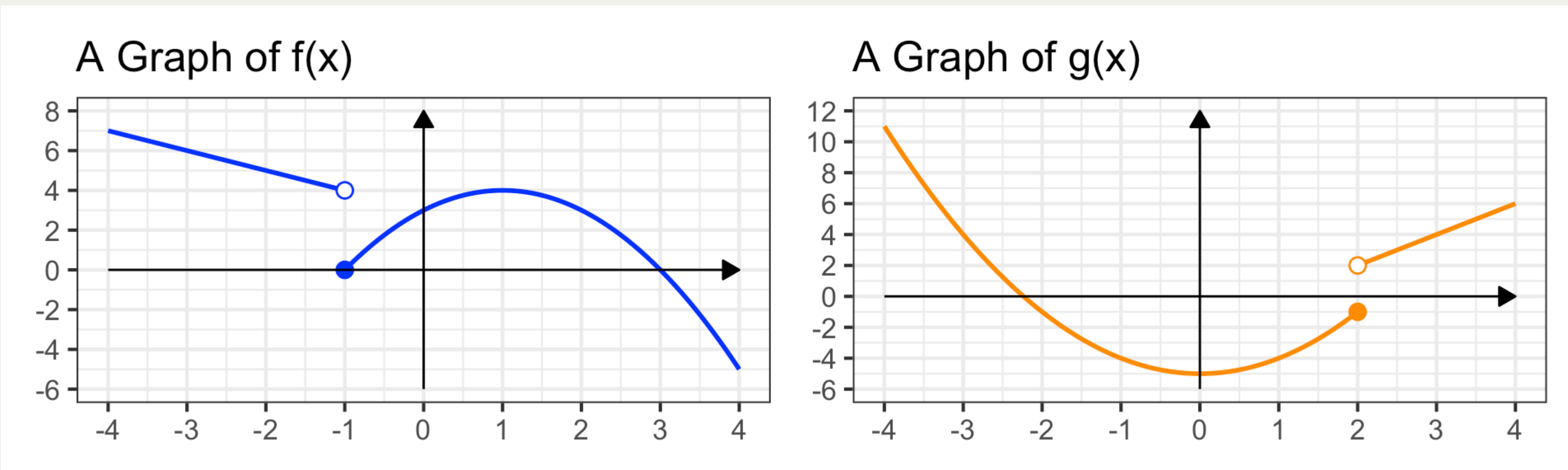

Solution. Working from the inside out in each case:

- $f(g(f(2))) = f(g(3)) = f(4) = \boxed{-5}$

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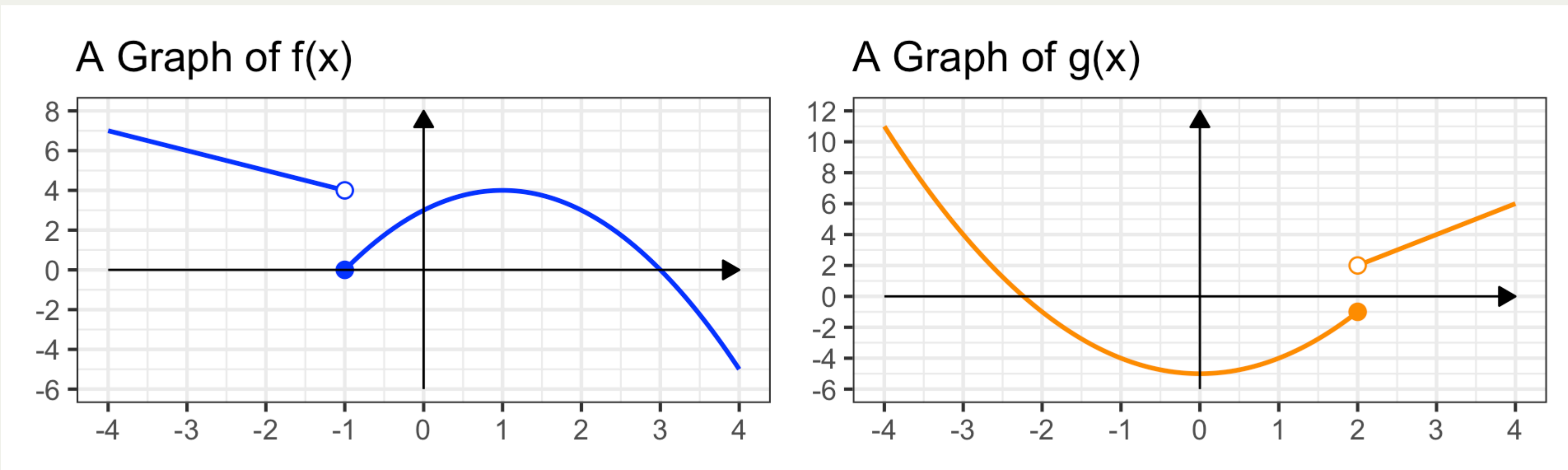


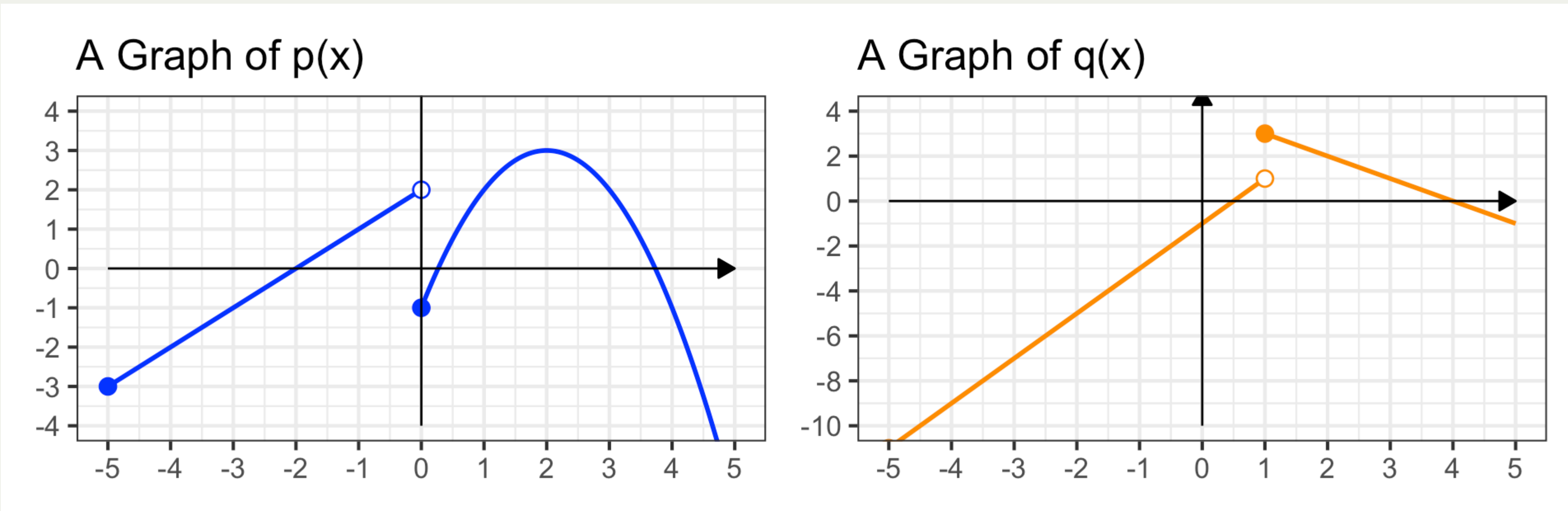
Solution. Working from the inside out in each case:

- $g(f(g(2))) = g(f(-1)) = g(0) = \boxed{-5}$

Composition from Graphs Practice

Try It! 3: The graphs of $p(x)$ and $q(x)$ appear below. Use them to evaluate each of the following.

$(a)\ \ (p \circ q)(2)$ $(b)\ \ q(p(-2))$ $(c)\ \ (q \circ p)(0)$ $(d)\ \ (p \circ p)(3)$ $(e)\ \ (q \circ q)(-1)$ $(f)\ \ p(q(p(2)$



Applied Problems

Applied Problem 1: A forest fire leaves behind an expanding circular burn. The radius is increasing at rate $r(t) = 2t + 1$ (feet, t in minutes).

Express the **area burned** as a function of time t, and find the total area burned after 10 minutes.

Solution. The area of a circle is $A = \pi r^2$, so $A(r) = \pi r^2$. We substitute $r(t)$ in place of r:

$$A(r(t)) = \pi(r(t))^2$$

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$$\begin{aligned} A(r(t)) &= \pi (r(t))^2 \\ &= \pi (2t+1)^2 \end{aligned}$$

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$$\begin{aligned} A(r(t)) &= \pi (r(t))^2 \\ &= \pi (2t+1)^2 \\ &= \pi \left(4t^2 + 4t + 1\right) \end{aligned}$$

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$$\begin{aligned} A(r(t)) &= \pi(r(t))^2 \\ &= \pi(2t+1)^2 \\ &= \pi\left(4t^2 + 4t + 1\right) \end{aligned}$$

After 10 minutes: $A(r(10)) = \pi(4(100) + 4(10) + 1) = \boxed{441\pi \approx 1385.4 \text{ sq. ft.}}$

Applied Problems (Cont’d)

Applied Problem 2: The number of bacteria in a refrigerated food product is given by

$$N\left(T\right) = 23T^2 - 56T + 1 \qquad 3 < T < 33$$

where T is the temperature (°F). After the food is removed from the refrigerator, the temperature is given by $T\left(t\right) = 5t + 1.5$, where t is time in hours.

(a) Find the composite function $N\left(T\left(t\right)\right)$.

(b) Find the time (to two decimal places) when the bacteria count reaches $6{,}752$.

Solution to (a). Substitute $T\left(t\right) = 5t + 1.5$ into $N\left(T\right)$:

$$N\left(T\left(t\right)\right) = 23(5t + 1.5)^2 - 56\left(5t + 1.5\right) + 1$$

Applied Problems (Cont'd)

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Solution to (a). Substitute $T(t) = 5t + 1.5$ into $N(T)$:

$$\begin{aligned} N(T(t)) &= 23(5t + 1.5)^2 - 56(5t + 1.5) + 1 \\ &= 23\left(25t^2 + 15t + 2.25\right) - 280t - 84 + 1 \end{aligned}$$

Applied Problems (Cont'd)

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$$\begin{aligned} N(T(t)) &= 23(5t + 1.5)^2 - 56(5t + 1.5) + 1 \\ &= 23\left(25t^2 + 15t + 2.25\right) - 280t - 84 + 1 \\ &= 575t^2 + 345t + 51.75 - 280t - 83 \end{aligned}$$

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Solution to (a). Substitute $T(t) = 5t + 1.5$ into $N(T)$:

$$\begin{aligned} N(T(t)) &= 23(5t + 1.5)^2 - 56(5t + 1.5) + 1 \\ &= 23\left(25t^2 + 15t + 2.25\right) - 280t - 84 + 1 \\ &= 575t^2 + 345t + 51.75 - 280t - 83 \\ &= \boxed{575t^2 + 65t - 31.25} \end{aligned}$$

Applied Problems (Cont'd)

Applied Problem 2 (Cont'd): Find the time when $N\left(T\left(t\right)\right) = 6{,}752$.

Solution to (b). Set $575t^2 + 65t - 31.25 = 6{,}752$ and solve.

Quadratic Formula

As a reminder, one way to solve a quadratic equation of the form $ax^2 + bx + c = 0$ is via the *quadratic formula*, which yields $x = \dfrac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$.

$$575t^2 + 65t - 31.25 = 6752$$

Applied Problems (Cont’d)

Applied Problem 2 (Cont’d): Find the time when $N(T(t)) = 6{,}752$.

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$$
\begin{aligned}
& 575t^2 + 65t - 31.25 = 6752 \\
\implies\ & 575t^2 + 65t - 6783.25 = 0
\end{aligned}
$$

Applied Problems (Cont'd)

Applied Problem 2 (Cont'd): Find the time when $N(T(t)) = 6{,}752$.

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$$\begin{aligned} 575t^2 + 65t - 31.25 &= 6752 \\ \implies 575t^2 + 65t - 6783.25 &= 0 \\ \implies t &= \frac{-65 \pm \sqrt{65^2 + 4(575)(6783.25)}}{2(575)} \end{aligned}$$

Applied Problems (Cont'd)

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>
> As a reminder, one way to solve a quadratic equation of the form $ax^2 + bx + c = 0$ is via the *quadratic formula*, which yields $x = \dfrac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$.

$$\begin{aligned} 575t^2 + 65t - 31.25 &= 6752 \\ \implies 575t^2 + 65t - 6783.25 &= 0 \\ \implies t &= \frac{-65 \pm \sqrt{65^2 + 4(575)(6783.25)}}{2(575)} \\ &= \frac{-65 \pm \sqrt{4225 + 15{,}601{,}475}}{1150} \\ &\approx \frac{-65 \pm 3950.5}{1150} \end{aligned}$$

Taking the positive root (since $t \geq 0$): $t \approx \dfrac{3885.5}{1150} \approx \boxed{3.38 \text{ hours}}$

Exit Ticket Task

Navigate to our MAT142 Exit Ticket Form, answer the questions, and complete the *task* below.

Note. Today's discussion is listed as `5. Composition of Functions`

Task: Consider $f(x) = 2x - 1$ and $g(x) = x^2 + 3$.

(a) Find $(f \circ g)(x)$ and evaluate at $x = 2$.

(b) Find $(g \circ f)(x)$ and evaluate at $x = 2$.

Summary and Next Time...

Ideas From Today

- The **composition** $(f \circ g)(x) = f(g(x))$ chains two functions together – g acts first, then f acts on the result.
- **Order matters:** $(f \circ g)(x)$ and $(g \circ f)(x)$ are generally different functions.
- We can evaluate compositions **numerically** (step by step) or **algebraically** (substitute $g(x)$ into f) – both give the same result.
- We can evaluate compositions **from graphs** by reading output values and using them as the next input.
- Composition arises naturally in applied settings whenever one quantity depends on another that is itself changing.

Looking Ahead

- Try It! 2 previewed something important. When $(f \circ g)(x) = x$ and $(g \circ f)(x) = x$, we say f and g are *inverses* of each other.
- That idea – *invertibility* – is something we'll encounter later in our course.

Next Time:

Exam I

Homework:

Complete Homework 4 on MyOpenMath