MAT 142: Limits and Rates of Change

Dr. Gilbert

June 2, 2026

Reminders

At our last meeting, we discussed each of the following:

- The domain of a function as the collection of all its permissible inputs.
- Domain restrictions resulting from denominators and even-index radicals.
    - Determining a function's domain algebraically.
- Identifying domain and range from a graph.

Try the following warm-up problems.

Problem 1: Write the domain of the function $f\,(x) = \dfrac{x^2 - 6x + 5}{x - 1}$ using interval notation

Problem 2: Write the domain of the function $g\,(x) = \dfrac{x}{\sqrt{\left(x^2 - 25\right)}}$ using interval notation.

Problem 3: Evaluate $h\,(5)$ for $f\,(x) = \dfrac{x^2 - 6x + 5}{x - 1}$

Problem 4: Evaluate $f\,(1)$ for $j\,(x) = \dfrac{x^2 - 6x + 5}{x - 1}$

Problem 5: Evaluate $k\,(0)$ and $k\,(2)$ for $k\,(x) = \dfrac{x^2 - 6x + 5}{x - 1}$.

Can you approximate the rate of change of $k\,(x)$ between $x = 0$ and $x = 2$?

Motivating Application

Near the surface of the moon, the distance a falling object travels is given by

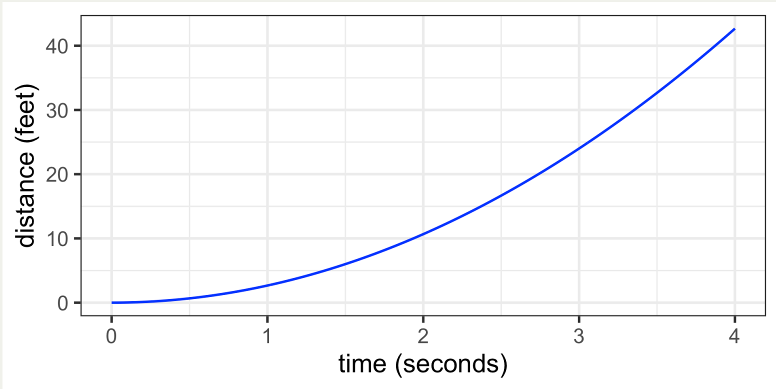
$$d(t) = 2.6667t^2$$

where t is in seconds and $d(t)$ is in feet.

Approximate the **velocity** of the object at time $t = 1.5$ seconds by using the average velocity from $t = 1$ to $t = 2$.

Before we work through this, consider the following questions:

- Does this object fall at a constant rate?
- You've discussed slopes of *linear* functions before. Do non-linear functions have slope?
- What strategy might help us approximate an "instantaneous" rate of change?



We'll revisit this application at the end of today's class.

Objectives

Today's class has two connected halves.

First half – Limits:

- Develop the notion of the *limit* as a generalization of function evaluation.
- Use limits to examine function behavior at points of discontinuity.
- Use limits to examine end behavior as $x \to \infty$ or $x \to -\infty$.

Second half – Rates of Change:

- Use the slope of the *secant line* to approximate the rate of change of a function at a point.
- Connect limits to the secant line approximation to preview the *derivative* from Calculus.

Limits of Functions

Definition (Limit): We say that the limit of $f(x)$ as x approaches a is equal to L, and write

$$\lim_{x \to a} f(x) = L$$

if the output values of $f(x)$ get **arbitrarily close to** L as x gets close to a.

A key feature of limits: we care about what the function is *approaching*, not what it equals *exactly* at $x = a$.

Check out these interactive Desmos plots for the definition in action.

- Demo 1
- Demo 2
- Demo 3
- Demo 4

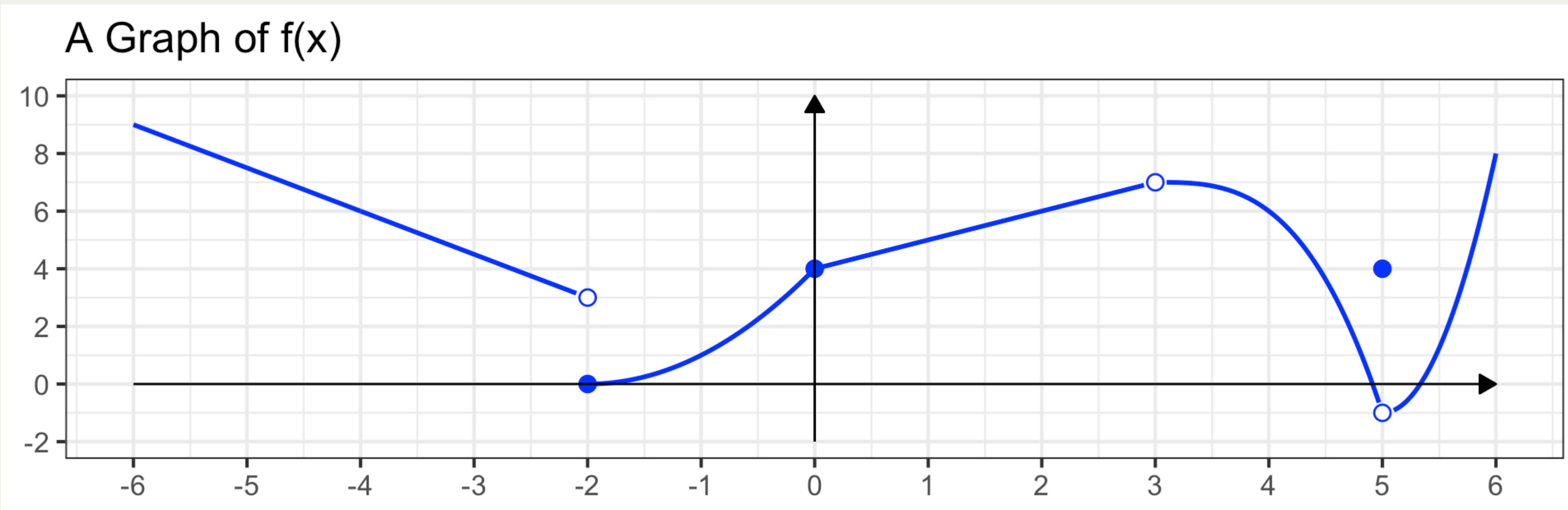
Shrink the value of "epsilon" (E) and notice whether the function output values (the y-values) stay within the horizontal tube.

Demo 4 shows what it looks like when a limit does not exist.

Completed Example 1

Problem: Consider the function $f(x)$ whose graph appears below. Determine each of the following limits, if they exist.

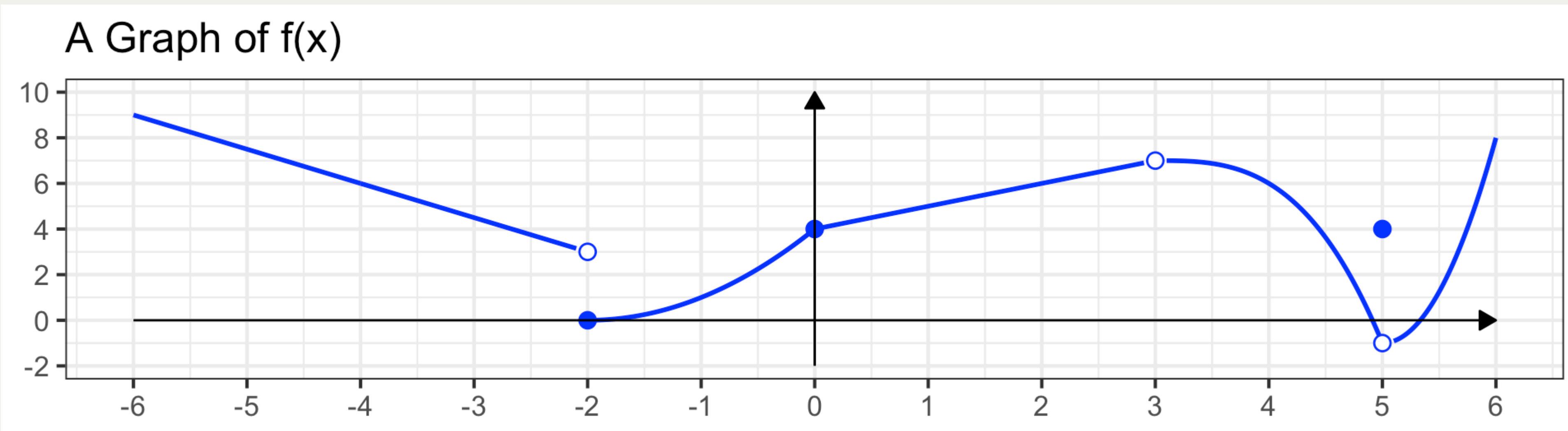
(a) $\lim_{x \to -4} f(x)$ (b) $\lim_{x \to -2} f(x)$ (c) $\lim_{x \to 0} f(x)$ (d) $\lim_{x \to 3} f(x)$ (e) $\lim_{x \to 5} f(x)$



Completed Example 1

Problem: Consider the function $f(x)$ whose graph appears below. Determine each of the following limits, if they exist.

(a) $\lim_{x\to -4} f(x)$ (b) $\lim_{x\to -2} f(x)$ (c) $\lim_{x\to 0} f(x)$ (d) $\lim_{x\to 3} f(x)$ (e) $\lim_{x\to 5} f(x)$

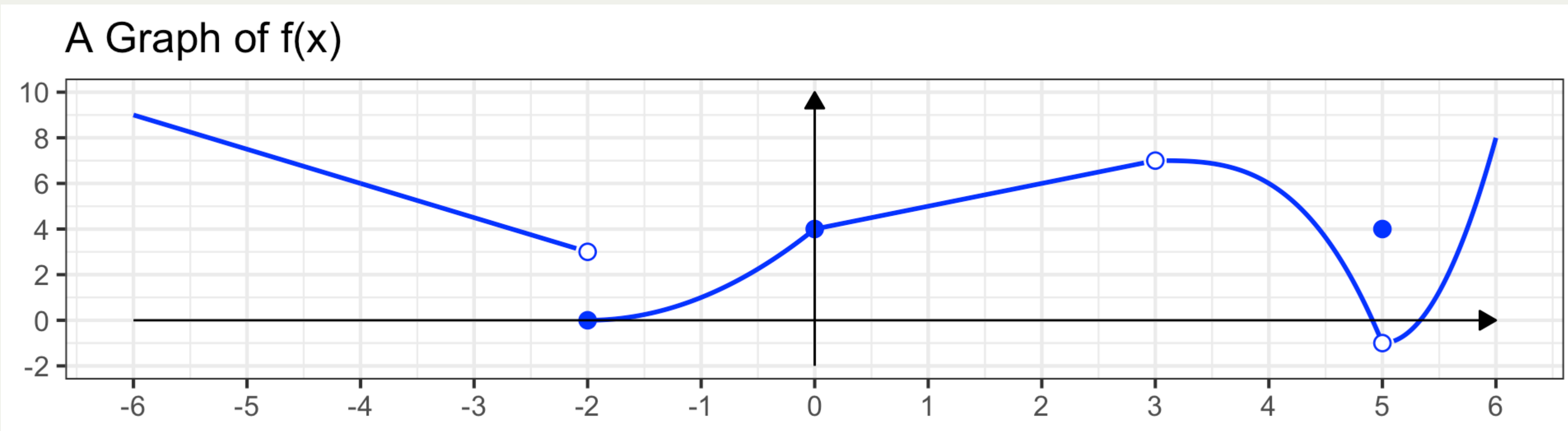


Solution.

Completed Example 1

Problem: Consider the function f (x) whose graph appears below. Determine each of the following limits, if they exist.

(a) $\lim_{x\to -4} f(x)$ (b) $\lim_{x\to -2} f(x)$ (c) $\lim_{x\to 0} f(x)$ (d) $\lim_{x\to 3} f(x)$ (e) $\lim_{x\to 5} f(x)$

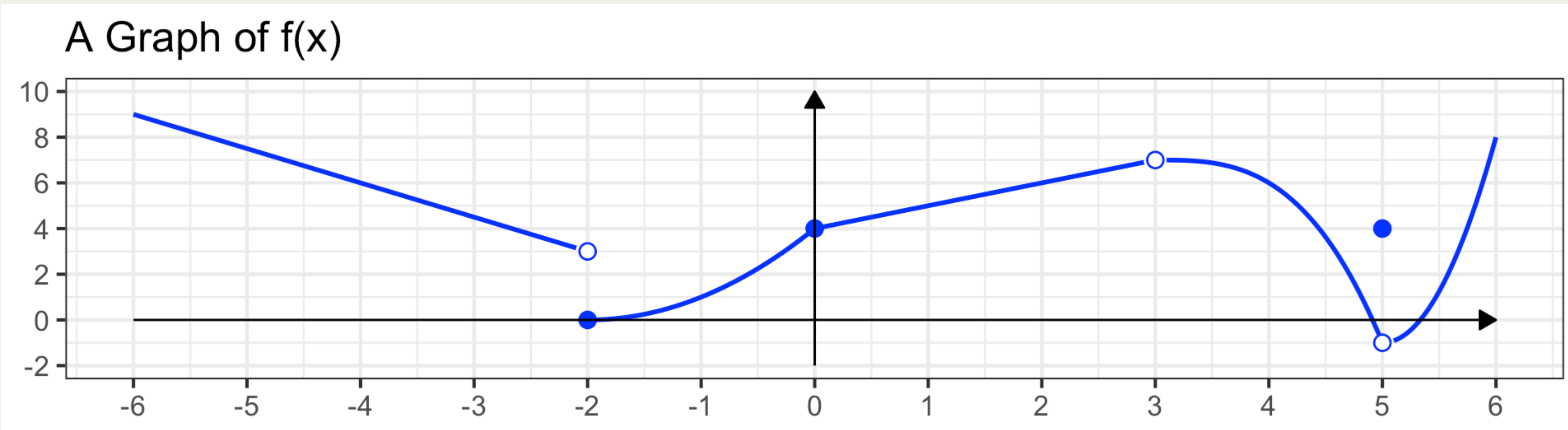


- $\lim_{x\to 3} f(x) = \boxed{7}$ – as $x \to 3$ from either side, $f(x) \to 7$, even though $f(3)$ is undefined. If we could assign $f(3) = 7$, the function would be *fixed* there.
- $\lim_{x\to 5} f(x) = \boxed{-1}$ – as $x \to 5$ from either side, $f(x) \to -1$, even though $f(5) = 4$. The limit does not ask what happens *at* $x = 5$, only *near* it.

Completed Example 1

Problem: Consider the function $f(x)$ whose graph appears below. Determine each of the following limits, if they exist.

(a) $\lim_{x\to-4} f(x)$ (b) $\lim_{x\to-2} f(x)$ (c) $\lim_{x\to0} f(x)$ (d) $\lim_{x\to3} f(x)$ (e) $\lim_{x\to5} f(x)$



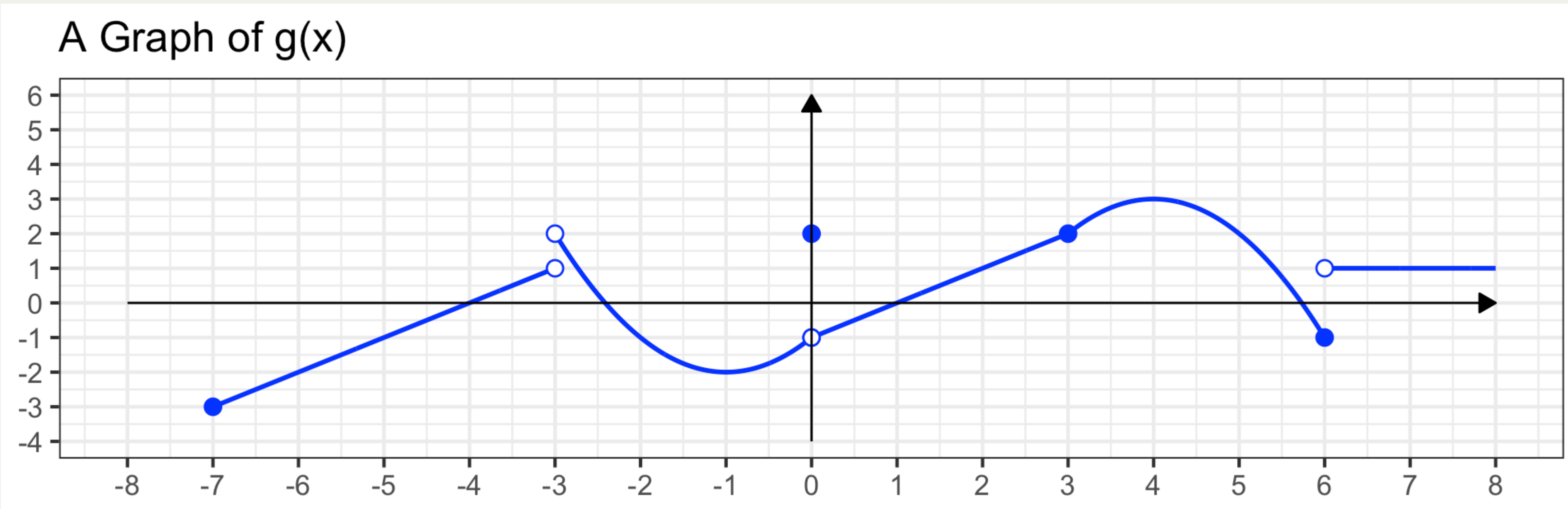
Key Takeaway

Limits provide a way to evaluate functions *near* a point, not necessarily *at* it. If a limit exists at a point of discontinuity, the discontinuity is **removable** – it could be fixed by simply reassigning the function value at that one point. If the limit does not exist, the discontinuity is more severe.

Limits from Graphs Practice I

Try It! 1: Consider the function $g(x)$ whose graph appears below. Determine each of the following limits, if they exist.

(a) $\lim\limits_{x \to -3} g(x)$ (b) $\lim\limits_{x \to 0} g(x)$ (c) $\lim\limits_{x \to 3} g(x)$ (d) $\lim\limits_{x \to 6} g(x)$



One-Sided Limits

Definition (One-Sided Limits): We can approach a limiting value from a single direction.

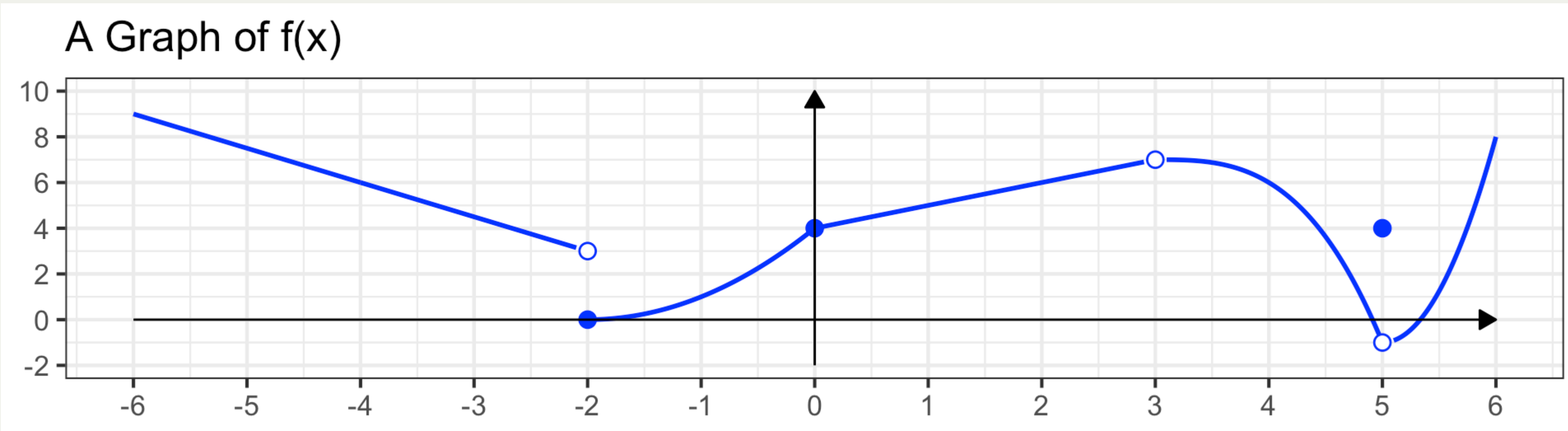
- The **left-hand limit** $\lim_{x \to a^-} f(x)$ describes what $f(x)$ approaches as x approaches a from values *less than* a.
- The **right-hand limit** $\lim_{x \to a^+} f(x)$ describes what $f(x)$ approaches as x approaches a from values *greater than* a.

Key connection: The two-sided limit $\lim_{x \to a} f(x)$ exists if and only if the left- and right-hand limits both exist *and are equal*.

Completed Example 2

Problem: Using the same graph of f (x), determine each of the following one-sided limits.

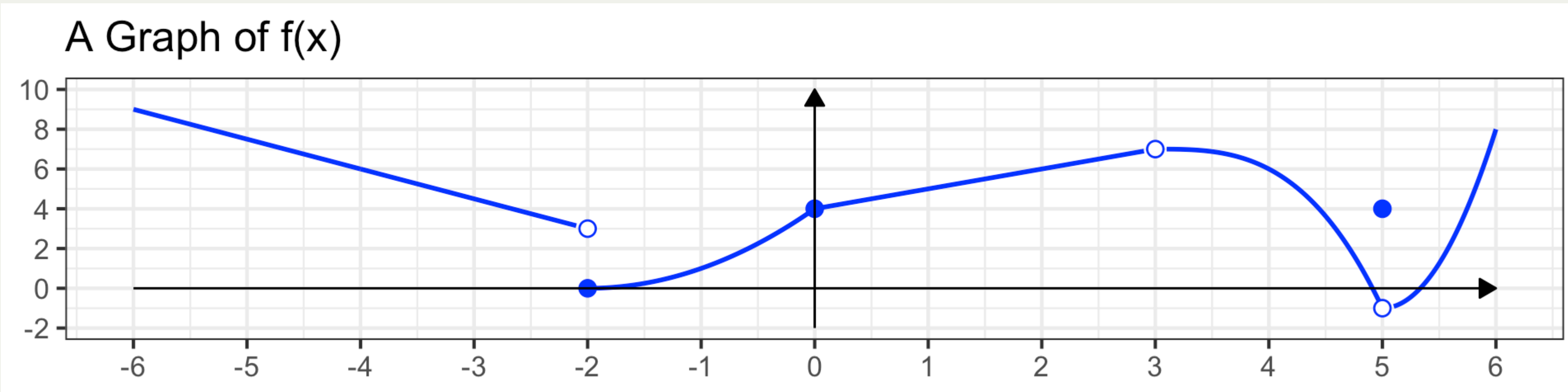
(a) $\lim_{x \to -6^+} f(x)$ (b) $\lim_{x \to -2^-} f(x)$ (c) $\lim_{x \to -2^+} f(x)$ (d) $\lim_{x \to 5^-} f(x)$ (e) $\lim_{x \to 5^+} f(x)$ (f) $\lim_{x \to 6^-} f(x)$



Completed Example 2

Problem: Using the same graph of f (x), determine each of the following one-sided limits.

(a) $\lim_{x\to -6^+} f(x)$ (b) $\lim_{x\to -2^-} f(x)$ (c) $\lim_{x\to -2^+} f(x)$ (d) $\lim_{x\to 5^-} f(x)$ (e) $\lim_{x\to 5^+} f(x)$ (f) $\lim_{x\to 6^-} f(x)$



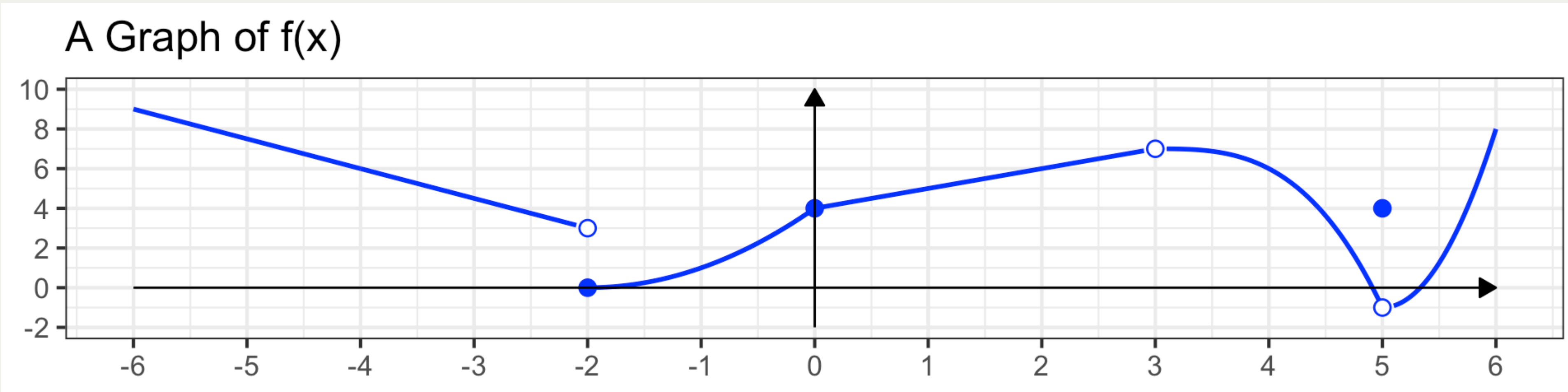
Solution.

- $\lim_{x\to -6^+} f(x) = \boxed{9}$ – approaching from the right, heights grow toward 9.
- $\lim_{x\to -2^-} f(x) = \boxed{3}$ – approaching from the left, heights drop toward 3.
- $\lim_{x\to -2^+} f(x) = \boxed{0}$ – approaching from the right, heights approach 0. Since $3 \neq 0$, $\lim_{x\to -2} f(x)$ does not exist.

Completed Example 2

Problem: Using the same graph of f (x), determine each of the following one-sided limits.

(a) $\lim_{x \to -6^+} f(x)$ (b) $\lim_{x \to -2^-} f(x)$ (c) $\lim_{x \to -2^+} f(x)$ (d) $\lim_{x \to 5^-} f(x)$ (e) $\lim_{x \to 5^+} f(x)$ (f) $\lim_{x \to 6^-} f(x)$



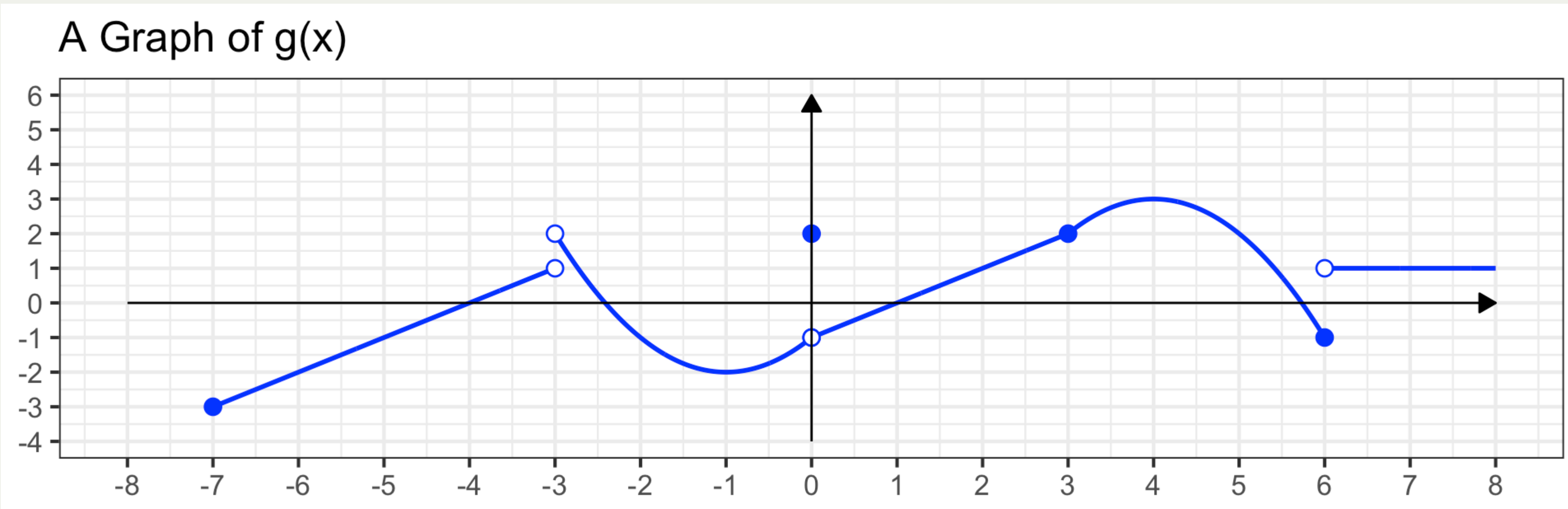
Solution.

- $\lim_{x \to 5^-} f(x) = \boxed{-1}$ and $\lim_{x \to 5^+} f(x) = \boxed{-1}$ – since these agree, $\lim_{x \to 5} f(x) = -1$.
- $\lim_{x \to 6^-} f(x) = \boxed{8}$.

Limits from Graphs Practice II

Try It! 2: Consider the function $g(x)$ whose graph appears below. Determine each of the following one-sided limits, if they exist.

(a) $\lim\limits_{x\to -7^+} g(x)$ (b) $\lim\limits_{x\to -3^-} g(x)$ (c) $\lim\limits_{x\to -3^+} g(x)$ (d) $\lim\limits_{x\to 6^-} g(x)$ (e) $\lim\limits_{x\to 6^+} g(x)$ (f) $\lim\limits_{x\to 8^-} g(x)$



Evaluating Limits Algebraically

Strategy: To evaluate $\lim_{x \to a} f(x)$ algebraically (for non-piecewise functions).

1. **Try direct substitution.** Plug $x = a$ into $f(x)$. If the result is defined, that's the limit.
2. **If substitution gives $\frac{0}{0}$**, try factoring and reducing to eliminate the discontinuity.
   - If this works, f has a **removable discontinuity** (a hole) at $x = a$, and the limit equals the reduced form evaluated at $x = a$.
   - If factoring doesn't help, more tools are needed (we'll develop them later).

Completed Example 3

Problem: Evaluate the following limits.

(a) $\lim_{x\to 3} 2x + 8$ (b) $\lim_{x\to 0} \dfrac{x-5}{x^2-25}$ (c) $\lim_{x\to -2} x^3 + 2x^2$

(d) $\lim_{x\to 5} \dfrac{x-5}{x^2-25}$ (e) $\lim_{x\to 2} \dfrac{x^2+3x-10}{x^2-5x+6}$ (f) $\lim_{x\to 7} \dfrac{x}{x-7}$

Solution. Parts (a), (b), and (c) can be evaluated by direct substitution.

$$\lim_{x\to 3} 2x + 8 = 2(3) + 8 = 6 + 8 = \boxed{14}$$

$$\lim_{x\to 0} \frac{x-5}{x^2-25} = \frac{0-5}{0^2-25} = \frac{-5}{-25} = \boxed{\frac{1}{5}}$$

$$\lim_{x\to -2} x^3 + 2x^2 = (-2)^3 + 2(-2)^2 = -8 + 8 = \boxed{0}$$

Completed Example 3 (Cont’d)

Problem: Evaluate $\lim\limits_{x \to 5} \dfrac{x-5}{x^2-25}$.

Solution. Direct substitution gives $\frac{0}{0}$, so we must factor and reduce.

$$\lim_{x \to 5} \frac{x-5}{x^2-25} = \lim_{x \to 5} \frac{x-5}{(x-5)(x+5)}$$

Completed Example 3 (Cont'd)

Problem: Evaluate $\lim_{x \to 5} \frac{x-5}{x^2-25}$.

Solution. Direct substitution gives $\frac{0}{0}$, so we must factor and reduce.

$$\begin{aligned}
\lim_{x \to 5} \frac{x-5}{x^2-25} &= \lim_{x \to 5} \frac{x-5}{(x-5)(x+5)} \\
&= \lim_{x \to 5} \frac{1}{x+5}
\end{aligned}$$

Completed Example 3 (Cont'd)

Problem: Evaluate $\lim_{x \to 5} \dfrac{x - 5}{x^2 - 25}$.

Solution. Direct substitution gives $\frac{0}{0}$, so we'll try to factor and reduce again.

$$\begin{aligned}
\lim_{x \to 5} \frac{x - 5}{x^2 - 25} &= \lim_{x \to 5} \frac{x - 5}{(x - 5)(x + 5)} \\
&= \lim_{x \to 5} \frac{1}{x + 5} \\
&= \frac{1}{5 + 5} = \boxed{\frac{1}{10}}
\end{aligned}$$

The function $f(x) = \dfrac{x - 5}{x^2 - 25}$ has a **removable discontinuity** (hole) at $x = 5$.

Completed Example 3 (Cont'd)

Problem: Evaluate $\lim_{x \to 2} \dfrac{x^2 + 3x - 10}{x^2 - 5x + 6}$.

Solution. Direct substitution gives $\frac{0}{0}$, so we'll try to factor and reduce again.

$$\lim_{x \to 2} \frac{x^2 + 3x - 10}{x^2 - 5x + 6} = \lim_{x \to 2} \frac{(x+5)(x-2)}{(x-3)(x-2)}$$

Completed Example 3 (Cont'd)

Problem: Evaluate $\lim_{x \to 2} \frac{x^2 + 3x - 10}{x^2 - 5x + 6}$.

Solution. Direct substitution gives $\frac{0}{0}$, so we'll try to factor and reduce again.

$$\begin{aligned} \lim_{x \to 2} \frac{x^2 + 3x - 10}{x^2 - 5x + 6} &= \lim_{x \to 2} \frac{(x+5)(x-2)}{(x-3)(x-2)} \\ &= \lim_{x \to 2} \frac{x+5}{x-3} \end{aligned}$$

Completed Example 3 (Cont'd)

Problem: Evaluate $\lim_{x \to 2} \frac{x^2 + 3x - 10}{x^2 - 5x + 6}$.

Solution. Direct substitution gives $\frac{0}{0}$, so we'll try to factor and reduce again.

$$\begin{aligned}
\lim_{x \to 2} \frac{x^2 + 3x - 10}{x^2 - 5x + 6} &= \lim_{x \to 2} \frac{(x+5)(x-2)}{(x-3)(x-2)} \\
&= \lim_{x \to 2} \frac{x+5}{x-3} \\
&= \frac{2+5}{2-3}
\end{aligned}$$

Completed Example 3 (Cont'd)

Problem: Evaluate $\lim_{x \to 2} \dfrac{x^2 + 3x - 10}{x^2 - 5x + 6}$.

Solution. Direct substitution gives $\frac{0}{0}$, so we'll try to factor and reduce again.

$$\begin{aligned}
\lim_{x \to 2} \frac{x^2 + 3x - 10}{x^2 - 5x + 6} &= \lim_{x \to 2} \frac{(x+5)(x-2)}{(x-3)(x-2)} \\
&= \lim_{x \to 2} \frac{x+5}{x-3} \\
&= \frac{2+5}{2-3} \\
&= \frac{7}{-1}
\end{aligned}$$

Completed Example 3 (Cont'd)

Problem: Evaluate $\lim\limits_{x\to 2} \dfrac{x^2 + 3x - 10}{x^2 - 5x + 6}$.

Solution. Direct substitution gives $\frac{0}{0}$, so we'll try to factor and reduce again.

$$\begin{aligned}
\lim_{x\to 2} \frac{x^2 + 3x - 10}{x^2 - 5x + 6} &= \lim_{x\to 2} \frac{(x+5)(x-2)}{(x-3)(x-2)} \\
&= \lim_{x\to 2} \frac{x+5}{x-3} \\
&= \frac{2+5}{2-3} \\
&= \frac{7}{-1} \\
&= \boxed{-7}
\end{aligned}$$

Completed Example 3 (Cont'd)

Problem: Evaluate $\lim_{x \to 7} \frac{x}{x-7}$.

Solution. Direct substitution gives $\frac{7}{0}$, which is undefined. Factoring offers no help – there is no factor of $(x-7)$ in the numerator to cancel.

We don't yet have the tools to evaluate this limit. Graphing $f(x) = \frac{x}{x-7}$ near $x = 7$ reveals that the function escapes to $\pm\infty$ – this is a **non-removable discontinuity** (a vertical asymptote).

> **Recap**
>
> - Direct substitution works when f is well-behaved at $x = a$.
> - When substitution gives $\frac{0}{0}$, factor and reduce – this reveals a removable discontinuity.
> - When the denominator is 0 but the numerator is not, we have a non-removable discontinuity. More tools will come later in our course.

Computing Limits Practice

Compute the following limits using algebraic techniques, if possible.

Try It! 3: $\lim_{x \to 2} 3x^2 - x + 5$

Try It! 4: $\lim_{x \to -1} \dfrac{x + 3}{x^2 + 1}$

Try It! 5: $\lim_{x \to 4} \sqrt{(x^2 - 16)}$

Try It! 6: $\lim_{x \to 3} \dfrac{x^2 - 9}{x - 3}$

Try It! 7: $\lim_{x \to -2} \dfrac{x^2 + 5x + 6}{x^2 - x - 6}$

Rates of Change

Recall: The slope of a line through (x_1, y_1) and (x_2, y_2) is $m = \dfrac{y_2 - y_1}{x_2 - x_1}$.

Definition (Secant Line): Given a function $f(x)$, the *secant line* through the points $(x_1, f(x_1))$ and $(x_2, f(x_2))$ has slope:

$$m = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$

The slope of the secant line gives the **average rate of change** of f between x_1 and x_2.

Check out this interactive Desmos plot – a function is graphed in blue and the secant line as a dashed black line.

Completed Example 4

Problem: Compute the slope of the secant line to $f(x) = -(x-3)^2 + 2$ at $x_1 = 1$ and $x_2 = 4$.

Solution. First, find the two points the secant line passes through.

$$f(1) = -(1-3)^2 + 2 = -4 + 2 = -2$$
$$f(4) = -(4-3)^2 + 2 = -1 + 2 = 1$$

The secant line passes through $(1, -2)$ and $(4, 1)$.

Now apply the slope formula.

$$m = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$

Completed Example 4

Problem: Compute the slope of the secant line to $f(x) = -(x-3)^2 + 2$ at $x_1 = 1$ and $x_2 = 4$.

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The secant line passes through $(1, -2)$ and $(4, 1)$.

Now apply the slope formula.

$$\begin{aligned} m &= \frac{f(x_2) - f(x_1)}{x_2 - x_1} \\ &= \frac{1 - (-2)}{4 - 1} \end{aligned}$$

Completed Example 4

Problem: Compute the slope of the secant line to $f(x) = -(x-3)^2 + 2$ at $x_1 = 1$ and $x_2 = 4$.

Solution. First, find the two points the secant line passes through.

$$f(1) = -(1-3)^2 + 2 = -4 + 2 = -2$$
$$f(4) = -(4-3)^2 + 2 = -1 + 2 = 1$$

The secant line passes through $(1, -2)$ and $(4, 1)$.

Now apply the slope formula.

$$\begin{aligned} m &= \frac{f(x_2) - f(x_1)}{x_2 - x_1} \\ &= \frac{1 - (-2)}{4 - 1} \\ &= \frac{3}{3} = \boxed{1} \end{aligned}$$

The slope of the secant line to $f(x) = -(x-3)^2 + 2$ between $x = 1$ and $x = 4$ is 1.

The Secant Line Near $x = a$

We get more flexibility by writing $x_1 = a$ and $x_2 = a + h$ for some small step size h. Then:

$$m = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$

The Secant Line Near $x = a$

We get more flexibility by writing $x_1 = a$ and $x_2 = a + h$ for some small step size h. Then:

$$\begin{aligned} m &= \frac{f(x_2) - f(x_1)}{x_2 - x_1} \\ &= \frac{f(a+h) - f(a)}{(a+h) - a} \end{aligned}$$

The Secant Line Near $x = a$

We get more flexibility by writing $x_1 = a$ and $x_2 = a + h$ for some small step size h. Then:

$$\begin{aligned} m &= \frac{f(x_2) - f(x_1)}{x_2 - x_1} \\ &= \frac{f(a+h) - f(a)}{(a+h) - a} \\ &= \frac{f(a+h) - f(a)}{h} \end{aligned}$$

This expression is called the **difference quotient** – you've already practiced computing $f(a + h)$.

Completed Example 5

Problem: Compute the slope of the secant line to $f(x) = -(x-3)^2 + 2$ near $x = 2$, using a step size of $h = 1$.

Solution.

$$m = \frac{f(a+h) - f(a)}{h}$$

Completed Example 5

Problem: Compute the slope of the secant line to $f(x) = -(x-3)^2 + 2$ near $x = 2$, using a step size of $h = 1$.

Solution.

$$\begin{aligned} m &= \frac{f(a+h) - f(a)}{h} \\ &= \frac{f(2+1) - f(2)}{1} \end{aligned}$$

Completed Example 5

Problem: Compute the slope of the secant line to $f(x) = -(x-3)^2 + 2$ near $x = 2$, using a step size of $h = 1$.

Solution.

$$\begin{aligned} m &= \frac{f(a+h) - f(a)}{h} \\ &= \frac{f(2+1) - f(2)}{1} \\ &= \frac{f(3) - f(2)}{1} \end{aligned}$$

Completed Example 5

Problem: Compute the slope of the secant line to $f(x) = -(x-3)^2 + 2$ near $x = 2$, using a step size of $h = 1$.

Solution.

$$\begin{aligned} m &= \frac{f(a+h) - f(a)}{h} \\ &= \frac{f(2+1) - f(2)}{1} \\ &= \frac{f(3) - f(2)}{1} \\ &= \frac{2-1}{1} = \boxed{1} \end{aligned}$$

Connecting Limits to Rates of Change

Question: What happens as we take h smaller and smaller in the secant line formula?

Use this interactive Desmos plot – grab the h slider and shrink it toward 0. What do you notice about the secant line as $h \to 0$?

As $h \to 0$, the secant line approaches the *tangent line* to the curve at $x = a$. This is a line that just “touches” the function at the single point $(A, f(a))$. Its slope is the *instantaneous rate of change*, or *derivative*, of f at $x = a$.

$$\text{Derivative of } f \text{ at } x = a: \qquad m = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

The *derivative* of a function is a central object of study in a first-semester course in Calculus.

Completed Example 6

Problem: Use the limit definition to find the derivative of $f(x) = 4x + 7$ at $x = 5$.

(*Note.* You might recognize that f is *linear* with slope 4, so our answer should be 4.)

Solution.

$$m = \lim_{h \to 0} \frac{f(5 + h) - f(5)}{h}$$

Completed Example 6

Problem: Use the limit definition to find the derivative of $f(x) = 4x + 7$ at $x = 5$.

(*Note.* You might recognize that f is *linear* with slope 4, so our answer should be 4.)

Solution.

$$\begin{aligned} m &= \lim_{h\to 0} \frac{f(5+h) - f(5)}{h} \\ &= \lim_{h\to 0} \frac{[4(5+h)+7] - [4(5)+7]}{h} \end{aligned}$$

Completed Example 6

Problem: Use the limit definition to find the derivative of $f(x) = 4x + 7$ at $x = 5$.

(*Note.* You might recognize that f is *linear* with slope 4, so our answer should be 4.)

Solution.

$$\begin{aligned} m &= \lim_{h\to 0} \frac{f(5+h) - f(5)}{h} \\ &= \lim_{h\to 0} \frac{[4(5+h)+7] - [4(5)+7]}{h} \\ &= \lim_{h\to 0} \frac{20 + 4h + 7 - 20 - 7}{h} \end{aligned}$$

Completed Example 6

Problem: Use the limit definition to find the derivative of $f(x) = 4x + 7$ at $x = 5$.

(*Note.* You might recognize that f is *linear* with slope 4, so our answer should be 4.)

Solution.

$$
\begin{aligned}
m &= \lim_{h \to 0} \frac{f(5+h) - f(5)}{h} \\
&= \lim_{h \to 0} \frac{[4(5+h)+7] - [4(5)+7]}{h} \\
&= \lim_{h \to 0} \frac{20 + 4h + 7 - 20 - 7}{h} \\
&= \lim_{h \to 0} \frac{4h}{h}
\end{aligned}
$$

Completed Example 6

Problem: Use the limit definition to find the derivative of $f(x) = 4x + 7$ at $x = 5$.

(*Note.* You might recognize that f is *linear* with slope 4, so our answer should be 4.)

Solution.

$$\begin{aligned} m &= \lim_{h\to 0} \frac{f(5+h) - f(5)}{h} \\ &= \lim_{h\to 0} \frac{[4(5+h)+7] - [4(5)+7]}{h} \\ &= \lim_{h\to 0} \frac{20 + 4h + 7 - 20 - 7}{h} \\ &= \lim_{h\to 0} \frac{4h}{h} \\ &= \lim_{h\to 0} 4 = \boxed{4} \end{aligned}$$

As expected, the derivative of the linear function $f(x) = 4x + 7$ is its slope, $m = 4$.

Slope, Secant, and Derivative Practice

Try It! 8: Compute the slope of the secant line to $f(x) = \sqrt{(x+1)}$ at $x = 3$ and $x = 8$.

Try It! 9: Compute the slope of the secant line to $f(x) = -x^2 + 5$ near $x = 1$ using a step size of $h = 1$.

Try It! 10: Use the limit definition of the derivative to find the slope of $f(x) = 10 - 8x$ near $x = 2$ as $h \to 0$.

Applied Problems

Applied Problem 1: A driver stopped at a gas station at exactly 3:40 p.m. and began pumping gas. At exactly 3:44 p.m. the tank was full – they had pumped 10.7 gallons. What is the average rate of flow of gasoline into the tank?

Applied Problem 2: Near the surface of the moon, the distance an object falls is given by $d(t) = 2.6667t^2$, where t is in seconds and $d(t)$ is in feet.

(a) Find the average velocity of the object from $t = 1$ to $t = 2$.

(b) Use your answer to approximate the velocity at $t = 1.5$ seconds. Is this a good approximation? How could you find a better one?

Exit Ticket Task

Navigate to our MAT142 Exit Ticket Form, answer the questions, and complete the *task* below.

Note. Today's discussion is listed as `4. Limits and Rates of Change`

Task: Evaluate $\lim_{x \to 3} \frac{x^2 - 9}{x - 3}$ and describe what the result tells you about the function $f(x) = \frac{x^2 - 9}{x - 3}$ near $x = 3$.

Summary and Next Time...

Ideas From Today

- $\lim_{x \to a} f(x) = L$ means $f(x)$ approaches L as x approaches a – the limit doesn't care what happens exactly *at* a.
- Left- and right-hand limits must agree for a two-sided limit to exist.
- **Direct substitution** works when f is well-behaved. **Factor and reduce** when substitution gives $\frac{0}{0}$.
- A removable discontinuity (hole) can be detected and "fixed" via limits. A non-removable discontinuity cannot.
- The **secant line slope** $\dfrac{f(a+h) - f(a)}{h}$ gives the average rate of change over an interval.
- As $h \to 0$, this becomes the **derivative** – the instantaneous rate of change. You'll use this constantly in Calculus.

Resources

- Demo 1, Demo 2, Demo 3, Demo 4 – limit definition in action (Desmos)
- Secant line explorer – secant line and $h \to 0$ (Desmos)

Next Time:

Linear Functions and Their Properties

Homework:

Complete Homework 3 on MyOpenMath