

MAT 142: Intervals and Introduction to Functions

Dr. Gilbert

June 1, 2026

Reminders

At our last meeting, we discussed each of the following:

- Factoring out a greatest common factor.
- Factoring quadratic expressions, including differences of squares.
- Adding and subtracting rational expressions using a common denominator.
- Simplifying complex rational expressions.
- Using factoring to reduce rational expressions.

Try the following warm-up problems.

Problem 1: Expand and simplify
 $(x^2 - 5x + 2)(2x - 3x^2)$

Problem 2: Simplify $\frac{x^3 - 7x^2 + 12x}{3x - x^2}$

Problem 3: Simplify $\frac{5}{x+3} - \frac{x-8}{x+2}$

Problem 4: Simplify $\frac{x - \frac{1}{4}}{\frac{1}{x} + 2}$

Objectives

Today, we'll work with two main topics: describing intervals of real numbers, and an introduction to functions and function notation.

After today's class meeting, you should be able to:

- Identify important landmarks from the graph of a function – roots, y-intercept, local maxima/minima, and intervals of increase/decrease.
- Use interval notation to describe intervals of real numbers.
- Define what is meant by a *function*.
- Identify and use function notation.
- Evaluate a function at a given input, including symbolic inputs.
- Evaluate a piecewise-defined function.

Intervals of Real Numbers

Before diving into interval notation, we'll build some intuition by reading information directly from function graphs.

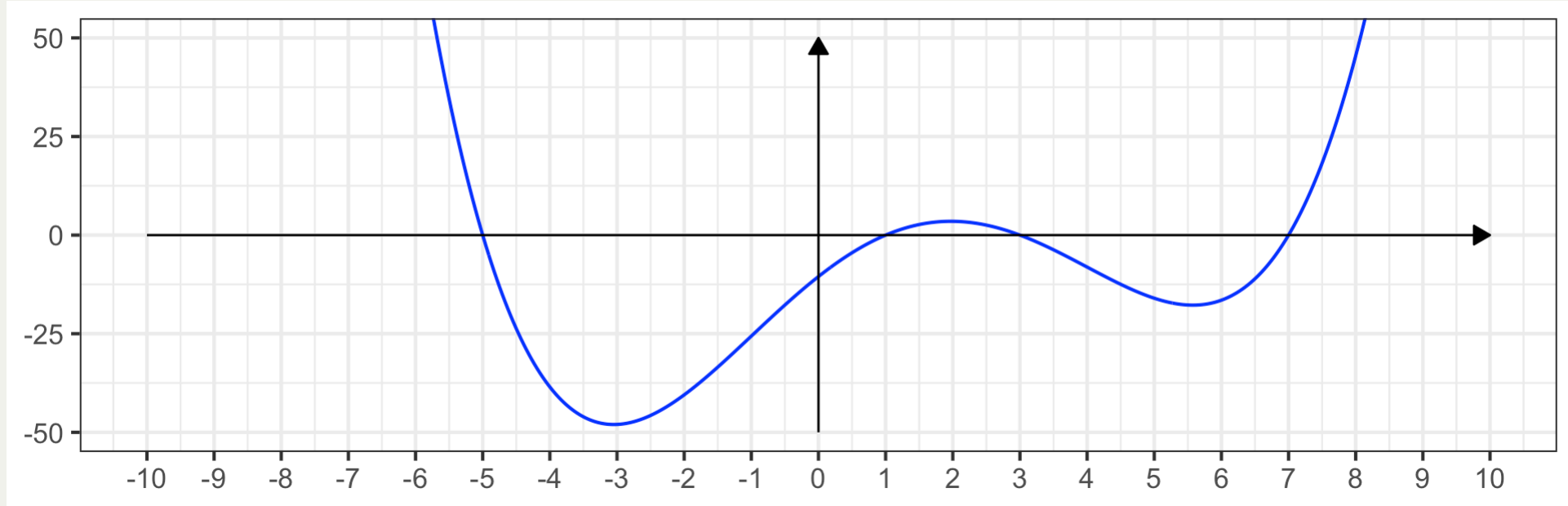
For any function we study, we'll want to be able to identify the following **landmarks**:

- **Roots** (x-intercepts): points on the graph where $y = 0$
- **y-intercept**: the point on the graph where $x = 0$
- **Local Maxima/Minima**: the x-coordinates of peaks and valleys
- **Intervals of Increase**: where the graph is rising (left to right)
- **Intervals of Decrease**: where the graph is falling (left to right)

Completed Example 1

Consider the function graphed on the right. Answer each of the following.

- Identify the locations of all roots (x-intercepts).
- Identify the y-intercept.
- Identify the x-coordinates of any local minima or maxima.
- Describe the interval(s) over which the function is increasing.
- Describe the interval(s) over which the function is decreasing.

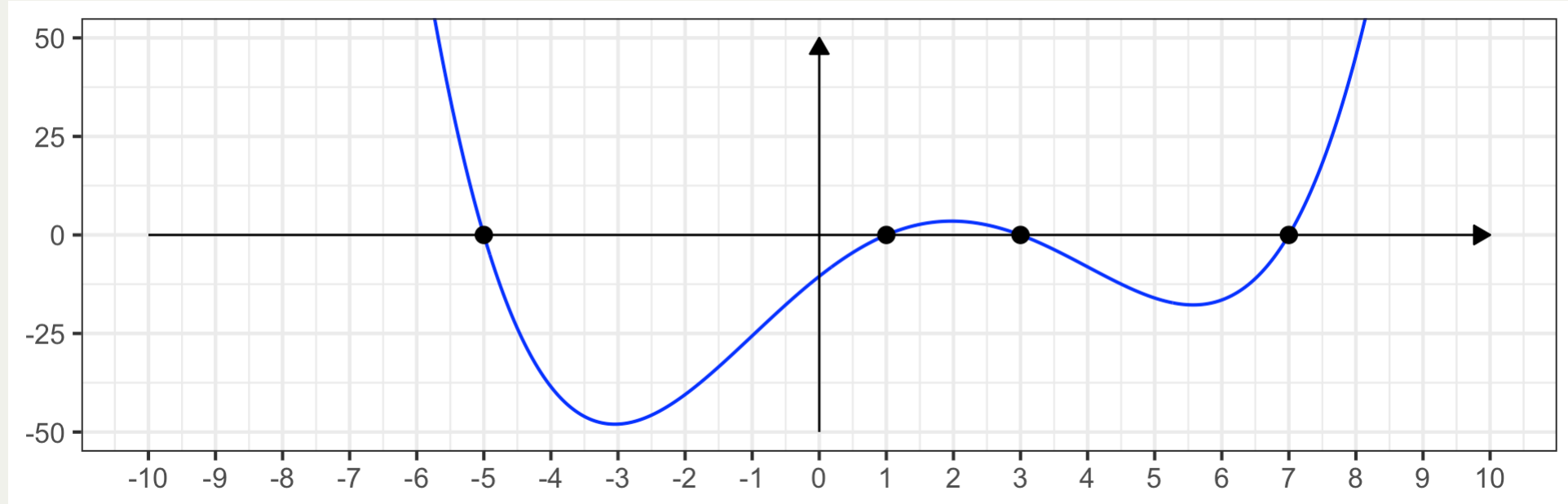


Solutions. By inspecting the graph (we'll develop exact methods later in the course).

Completed Example 1

Consider the function graphed on the right. Answer each of the following.

- **Roots:** approximately $(-5, 0)$, $(1, 0)$, $(3, 0)$, and $(7, 0)$
- Identify the y-intercept.
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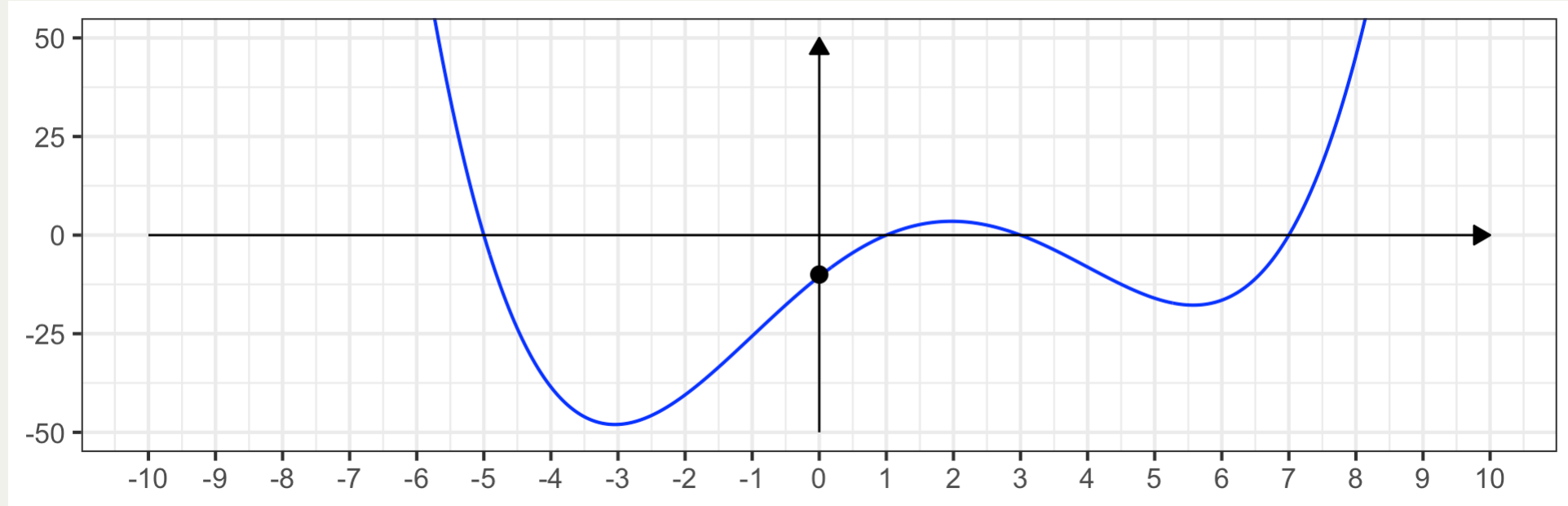


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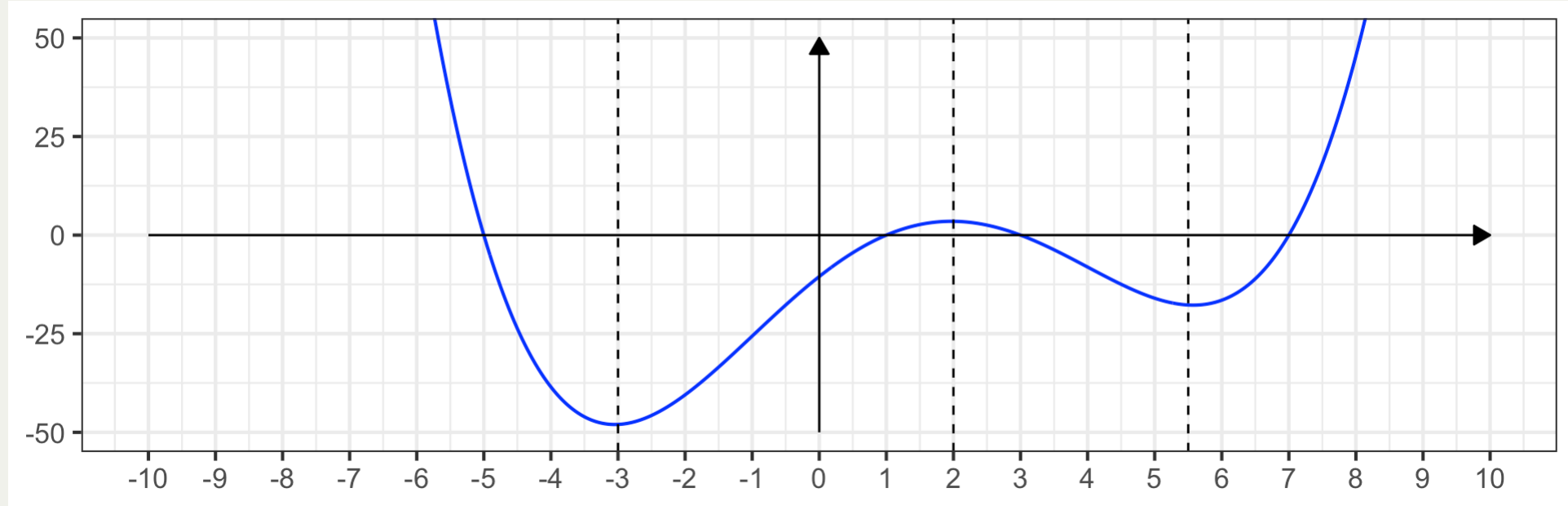


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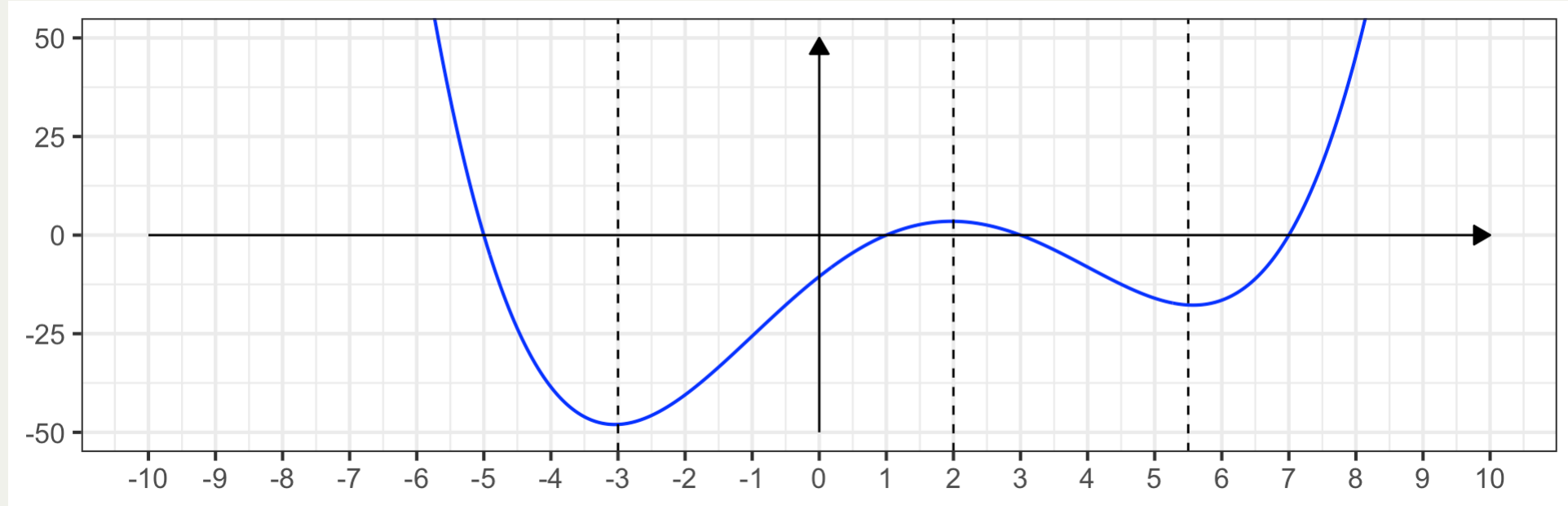


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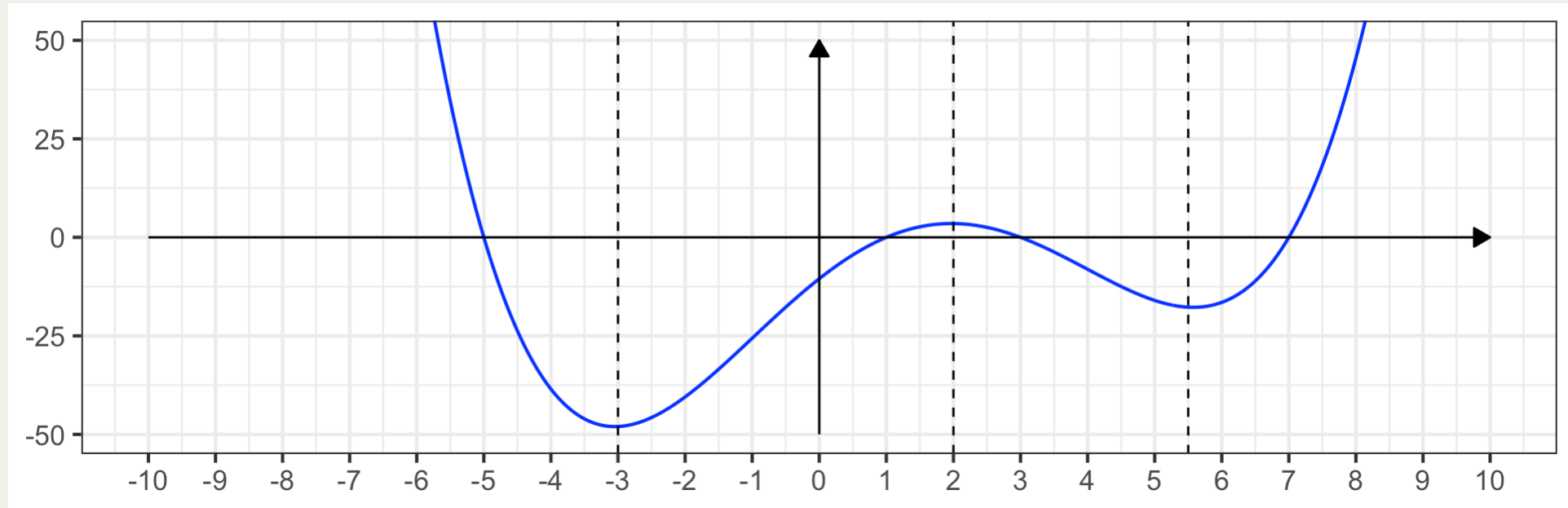


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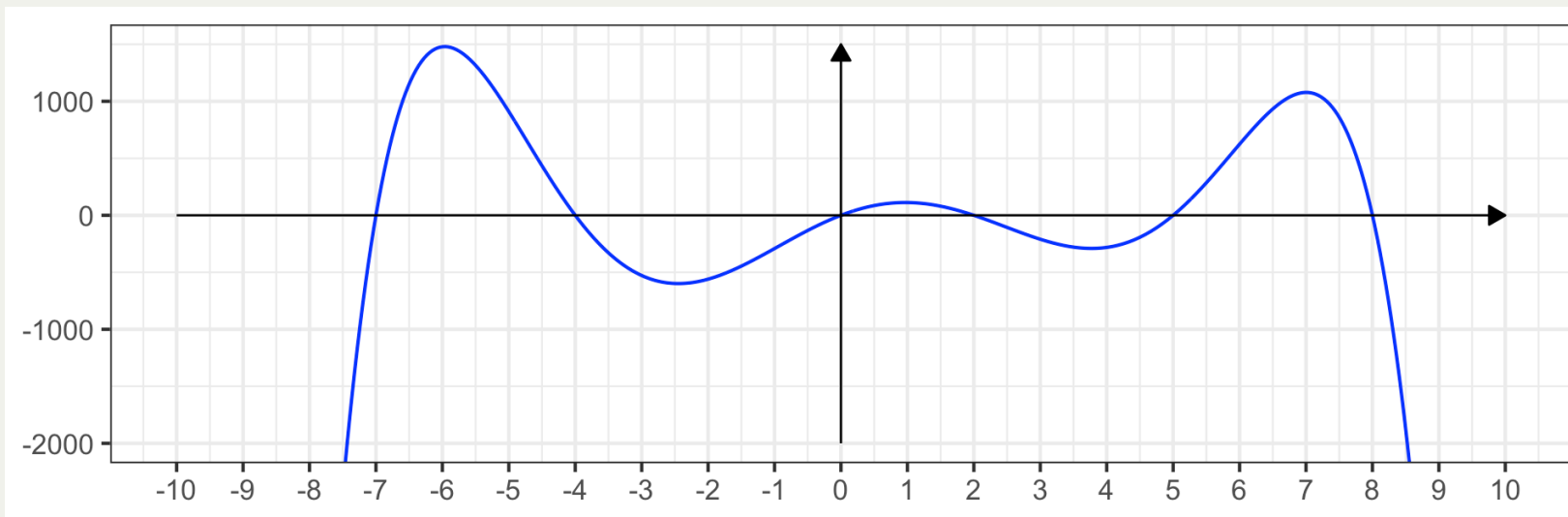


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Try It! 1

Consider the function graphed below. Answer each of the following.

- Identify the locations of all roots (x-intercepts).
- Identify the y-intercept.
- Identify the x-coordinates of any local minima or maxima.
- Describe the interval(s) over which the function is increasing.
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Interval Notation

It is useful to have a standard mathematical notation for describing intervals of real numbers. We'll make use of the following conventions.

Open intervals use parentheses and exclude the endpoints:

$$(a, b) = \text{all real numbers strictly between } a \text{ and } b$$

Closed intervals use square brackets and include the endpoints:

$$[a, b] = \text{all real numbers between } a \text{ and } b, \text{ including } a \text{ and } b$$

Half-open intervals mix the two:

$$(a, b] = \text{includes } b \text{ but not } a \qquad [a, b) = \text{includes } a \text{ but not } b$$



Overloaded Notation

The notation (a, b) is overloaded – it can mean an open interval *or* a point in the coordinate plane. Context makes the meaning clear.

Interval Notation (Cont'd)

The **union** operator $\cup$ lets us describe a collection of disjoint intervals:

$$(a, b) \cup (c, d) = \text{all real numbers in } (a, b) \text{ or in } (c, d)$$

We can chain as many unions together as needed.

For unbounded intervals, we use ∞ and $-\infty$. The unbounded end of an interval is always accompanied by parentheses (never brackets), since infinity is not a number that can be reached.

$$(-\infty, a) \qquad (a, \infty) \qquad (-\infty, \infty)$$

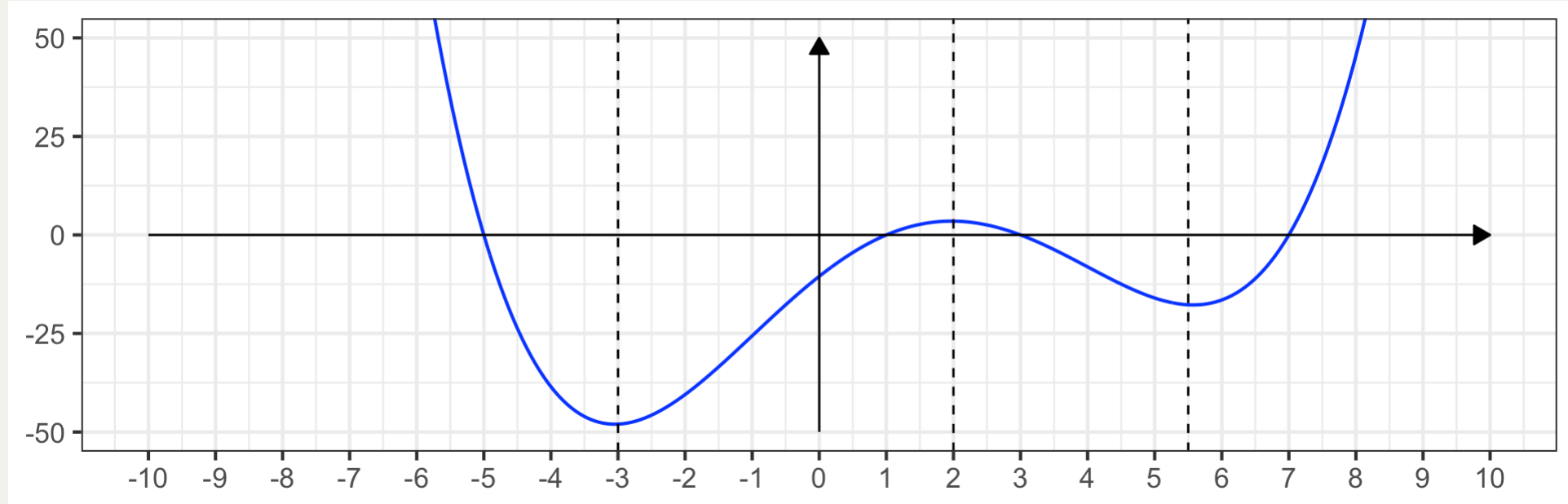
$$(-\infty, a] \qquad [a, \infty)$$

For a comprehensive review of interval notation, check out this [video from The Organic Chemistry Tutor](#).

Completed Example 2

Rewrite your answers below, using interval notation where appropriate.

- **Roots:** approximately $(-5, 0)$, $(1, 0)$, $(3, 0)$, and $(7, 0)$
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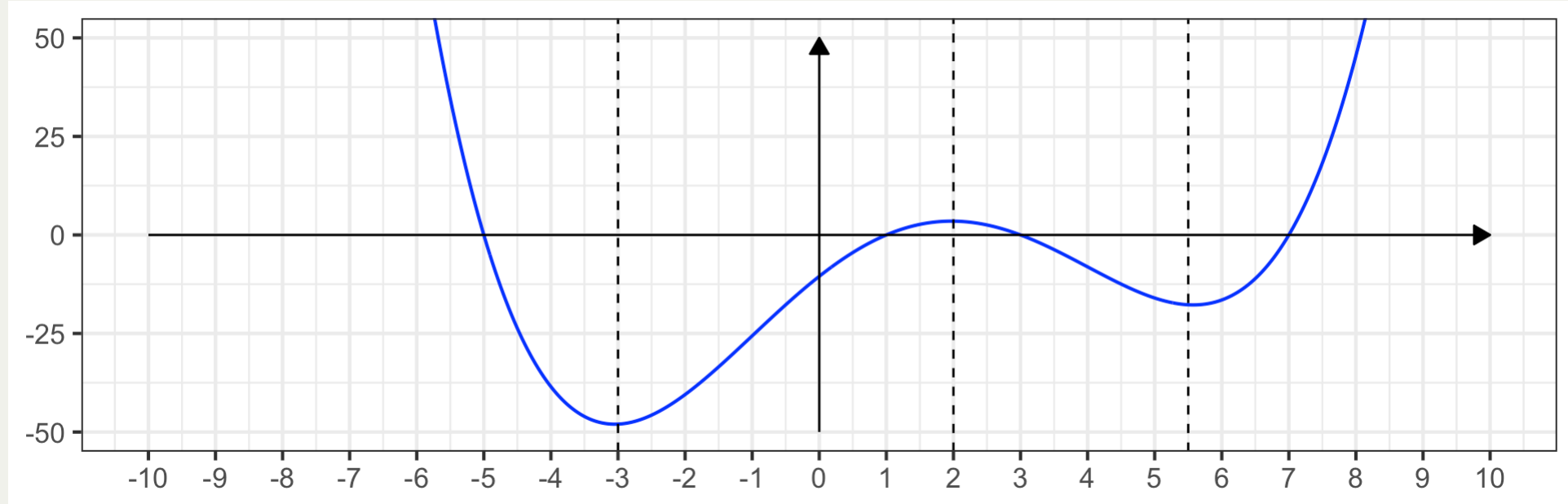


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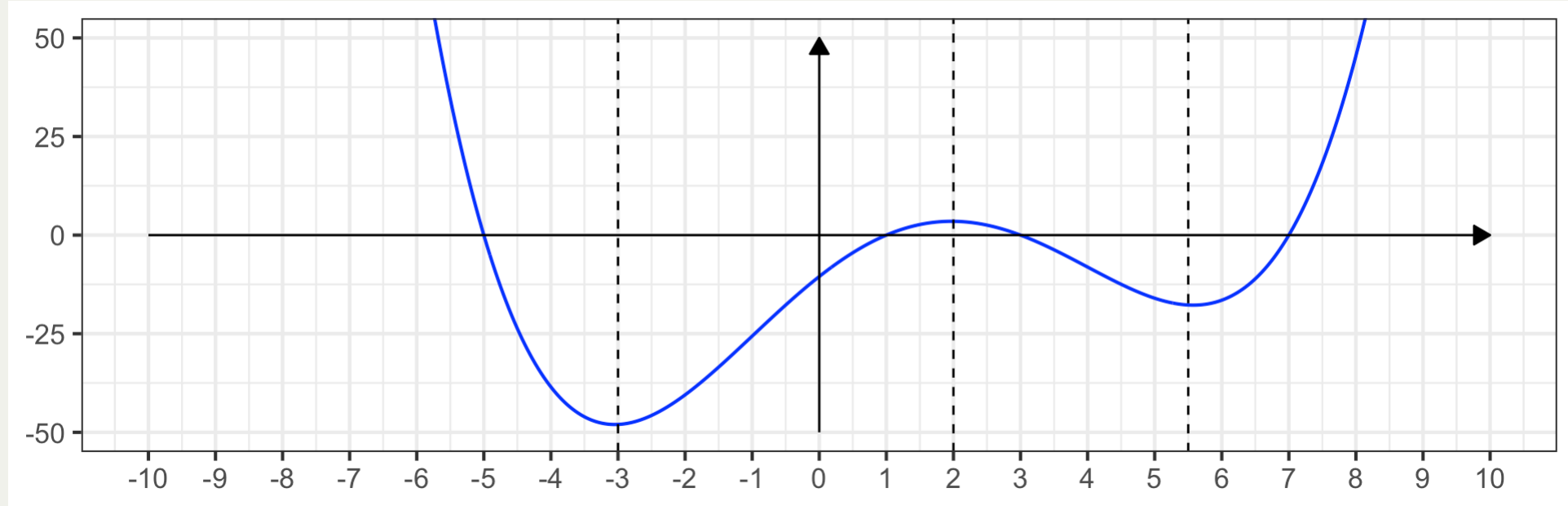


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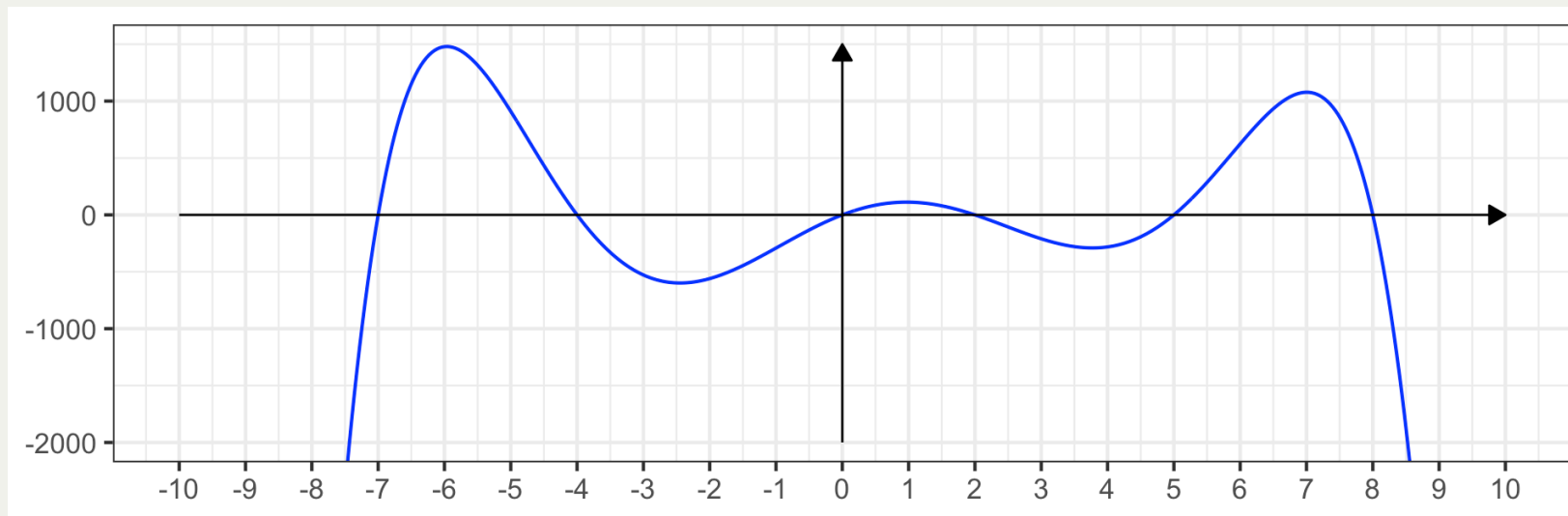


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Try It! 2

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Introduction to Functions

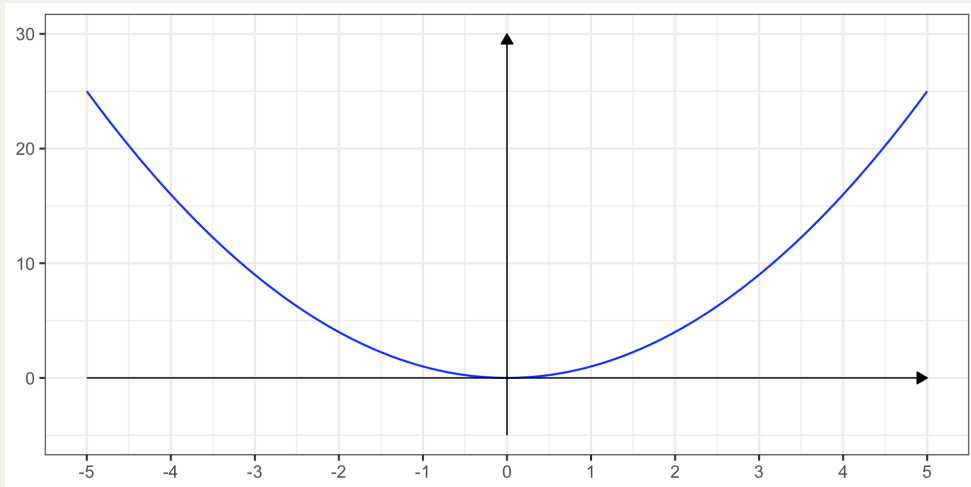
Definition (Function): A *function* is a rule that assigns **exactly one output** for any given input value from its *domain*.

In PreCalculus, we'll be interested in functions that take a real-valued input x and assign a real-valued output y .

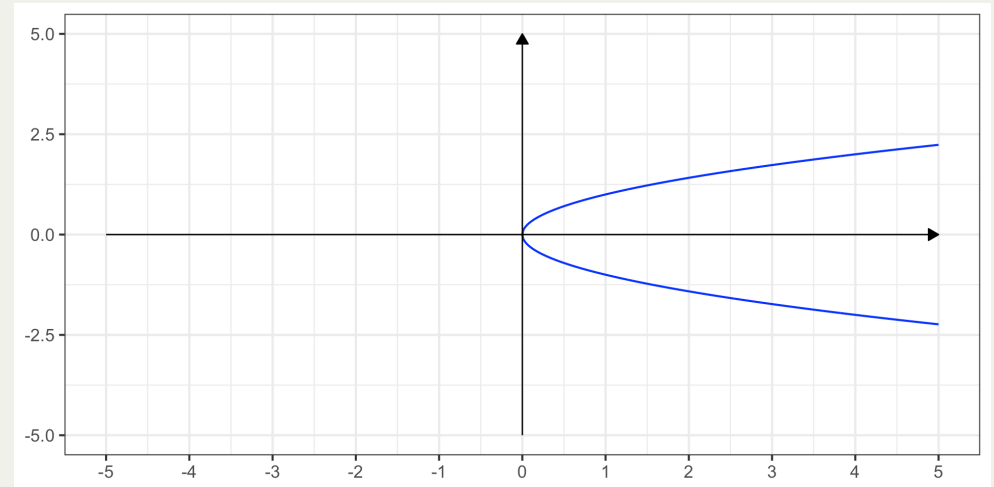
The key phrase is *exactly one*. For any input we plug in, there can only be one corresponding output.

The graphs below illustrate the difference between a function and a non-function:

A Function



Not A Function



Introduction to Functions

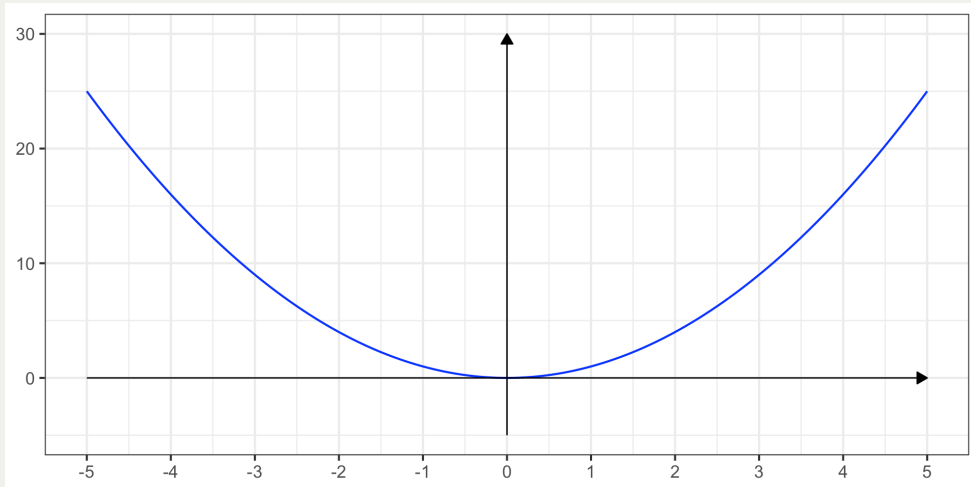
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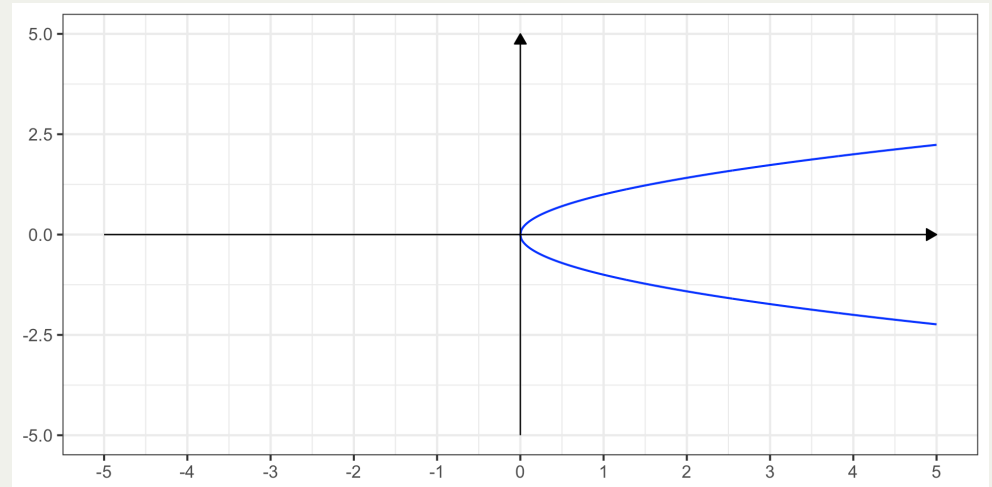
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A common method for testing whether a graph corresponds to a function is called the **vertical line test**. A graph passes (and is a function) if any vertical line drawn intersects the function at most once.

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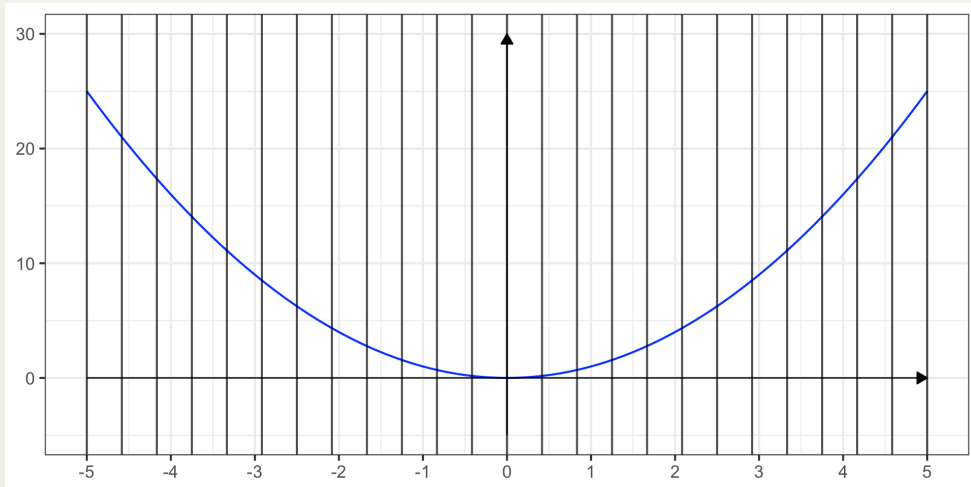
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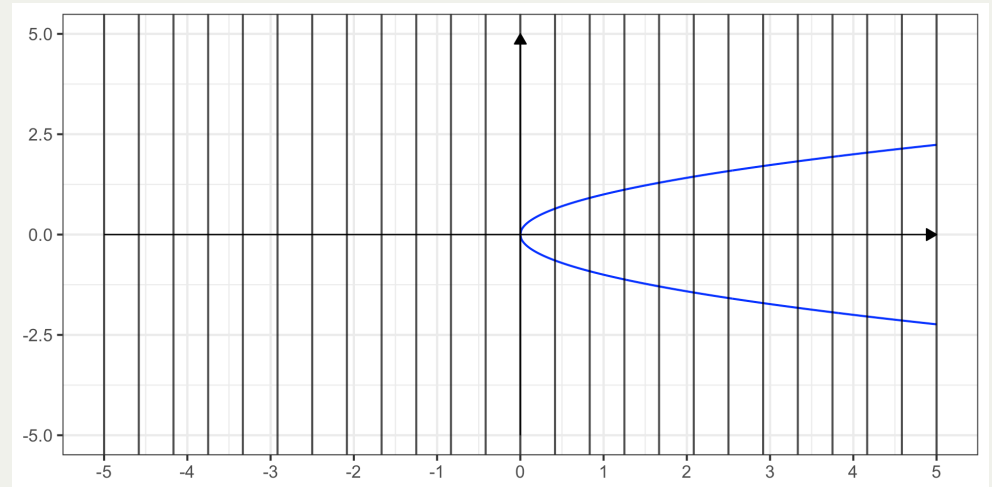
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Function Notation

Definition (Function Notation): We use *function notation* $f(x)$ to denote functions. This replaces “ $y =$ ” with “ $f(x) =$ ”.

For example, we can write $y = 5x + 4$ as $f(x) = 5x + 4$.

This notation is compact and informative – $f(x) = 5x + 4$ tells us exactly what happens to the input: *multiply by 5, then add 4*.

You can think of the function name f as the name of a machine, and $f(x)$ as the output of that machine when you feed it x :

Input: $x \rightarrow \boxed{f} \rightarrow$ Output: $f(x)$

We’re not limited to the name f – we can use g , h , j , k , or any other letter we like. The name is just a label for the rule.

Completed Example 3

Problem: Consider the function $f(x) = x^2 - 4x + 3$. Evaluate each of the following.

- (a) $f(2)$ (b) $f(0)$ (c) $f(-3)$ (d) $f(\pi)$ (e) $f(\heartsuit)$ (f) $f(x+h)$

Solution.

$$f(2) = (2)^2 - 4(2) + 3 = 4 - 8 + 3 = \boxed{-1}$$

$$f(0) = (0)^2 - 4(0) + 3 = \boxed{3}$$

$$f(-3) = (-3)^2 - 4(-3) + 3 = 9 + 12 + 3 = \boxed{24}$$

$$f(\pi) = (\pi)^2 - 4(\pi) + 3 = \boxed{\pi^2 - 4\pi + 3}$$

$$f(\heartsuit) = (\heartsuit)^2 - 4(\heartsuit) + 3 = \boxed{\heartsuit^2 - 4\heartsuit + 3}$$

Completed Example 3 (Cont'd)

Problem: Consider the function $f(x) = x^2 - 4x + 3$. Evaluate $f(x + h)$.

Solution. Replace every x with $(x + h)$:

$$f(x + h) = (x + h)^2 - 4(x + h) + 3$$

Completed Example 3 (Cont'd)

Problem: Consider the function $f(x) = x^2 - 4x + 3$. Evaluate $f(x + h)$.

Solution. Replace every x with $(x + h)$:

$$\begin{aligned} f(x + h) &= (x + h)^2 - 4(x + h) + 3 \\ &= (x + h)(x + h) - 4x - 4h + 3 \end{aligned}$$

Completed Example 3 (Cont'd)

Problem: Consider the function $f(x) = x^2 - 4x + 3$. Evaluate $f(x + h)$.

Solution. Replace every x with $(x + h)$:

$$\begin{aligned} f(x + h) &= (x + h)^2 - 4(x + h) + 3 \\ &= (x + h)(x + h) - 4x - 4h + 3 \\ &= x^2 + 2xh + h^2 - 4x - 4h + 3 \end{aligned}$$

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Problem: Consider the function $f(x) = x^2 - 4x + 3$. Evaluate $f(x + h)$.

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$$\begin{aligned} f(x + h) &= (x + h)^2 - 4(x + h) + 3 \\ &= (x + h)(x + h) - 4x - 4h + 3 \\ &= \boxed{x^2 + 2xh + h^2 - 4x - 4h + 3} \end{aligned}$$



Importance of $f(x + h)$

You'll see $f(x + h)$ often because it shows up in an extremely important quantity called the *difference quotient* in Calculus. Developing an ability to plug " $x + h$ " into a function, expand it, and simplify will be critical to your success.

We'll preview the *difference quotient* later in our course.

Piecewise-Defined Functions

Some functions behave differently on different intervals. We call these **piecewise-defined functions**.

Definition (Piecewise-Defined Function): A piecewise-defined function is defined by different rules on different pieces of its domain:

$$f(x) = \begin{cases} f_1(x) & ; \quad x \text{ in Interval 1} \\ f_2(x) & ; \quad x \text{ in Interval 2} \\ \vdots & \vdots \\ f_k(x) & ; \quad x \text{ in Interval } k \end{cases}$$

To evaluate a piecewise-defined function at a given input: **first determine which interval the input falls in, then apply the corresponding rule.**

Completed Example 4

Problem: Consider the piecewise-defined function

$$f(x) = \begin{cases} x^2 - 4 & ; \quad x < -2 \\ x + 5 & ; \quad -2 \leq x < 7 \\ \sqrt{x-7} + 12 & ; \quad x \geq 7 \end{cases}$$

Evaluate (a) $f(0)$, (b) $f(11)$, (c) $f(-5)$.

Solution.

(a) Since $x = 0$ satisfies the inequality $-2 \leq x < 7$, we use the second piece:

$$f(0) = 0 + 5 = \boxed{5}$$

(b) Since $x = 11$ satisfies $x \geq 7$, we use the third piece:

$$f(11) = \sqrt{11-7} + 12 = \sqrt{4} + 12 = 2 + 12 = \boxed{14}$$

(c) Since $x = -5$ satisfies $x < -2$, we use the first piece:

$$f(-5) = (-5)^2 - 4 = 25 - 4 = \boxed{21}$$

Functions Practice

Try each of the following practice problems.

Try It! 3: Consider $g(x) = 8 - 2x^3$. Describe in words how g transforms its input. Then evaluate $g(-1)$, $g(0)$, $g(2)$, $g(\sqrt{2})$, $g(\heartsuit)$, $g(\heartsuit + \diamondsuit)$, and $g(x + h)$.

Try It! 4: Consider $h(x) = \sqrt{x^2 + 9}$. Describe in words how h transforms its input. Then evaluate $h(5)$, $h(0)$, $h(-1)$, $h(\sqrt{2})$, $h(\heartsuit)$, and $h(x + h)$.

Try It! 5: Consider $j(x) = \frac{2x}{x^2 - 9}$. Describe in words how j transforms its input. Then evaluate $j(0)$, $j(5)$, $j(-3)$, $j(\sqrt{3})$, $j(\heartsuit)$, and $j(x + h)$.

Try It! 6: Consider $k(x) = \begin{cases} 8 - x^3 & ; \quad x \leq 2 \\ x & ; \quad x > 2 \end{cases}$. Evaluate $k(5)$, $k(0)$, and $k(2)$.

Applied Problem

Problem: The amount of garbage G (in tons per week) produced by a city with population p (in thousands of people) is given by $G = f(p)$.

(a) The town of Tola has a population of 40,000 and produces 13 tons of garbage per week. Express this information using function notation.

Solution. Since p is measured in thousands, 40,000 people corresponds to $p = 40$. We have:

$$f(40) = 13$$

(b) Explain the meaning of the statement $f(5) = 2$.

Solution. A city with a population of 5,000 people produces 2 tons of garbage per week.

Exit Ticket Task

Navigate to [our MAT142 Exit Ticket Form](#), answer the questions, and complete the *task* below.

Note. Today's discussion is listed as [2. Interval Notation and Functions](#)



Task: Consider the function $f(x) = 3x^2 - x + 5$. Evaluate $f(0)$, $f(-2)$, and $f(x + h)$.

Summary and Next Time...

Ideas From Today

- Read landmarks from a function graph: roots, y-intercept, local maxima/minima, increasing/decreasing intervals.
- Use **interval notation** (a, b) , $[a, b]$, $\cup$ to describe sets of real numbers.
- A **function** assigns exactly one output to each input – check with the Vertical Line Test.
- **Function notation** $f(x)$ replaces y and describes the transformation explicitly.
- Evaluate functions by substituting the input for every x , including symbolic inputs like $x + h$.
- **Piecewise functions** apply different rules on different intervals.

Resources

- [Interval notation review](#) – The Organic Chemistry Tutor

Next Time:

Domains and Ranges of Functions

Homework:

Start Homework 2 on MyOpenMath