# MAT 142: Right Triangle Trigonometry

Dr. Gilbert

June 12, 2026

# Reminders

Last class we studied the graphs of trigonometric functions, inverse trigonometric functions, and solved trigonometric equations. Try the following warm-up problems.

**Problem 1:** Evaluate $\arccos\left(\frac{-1}{2}\right)$ and $\arcsin\left(\frac{-1}{2}\right)$.

**Problem 2:** Find all solutions to $\sqrt{3}\tan(5x-3)+1=0$.

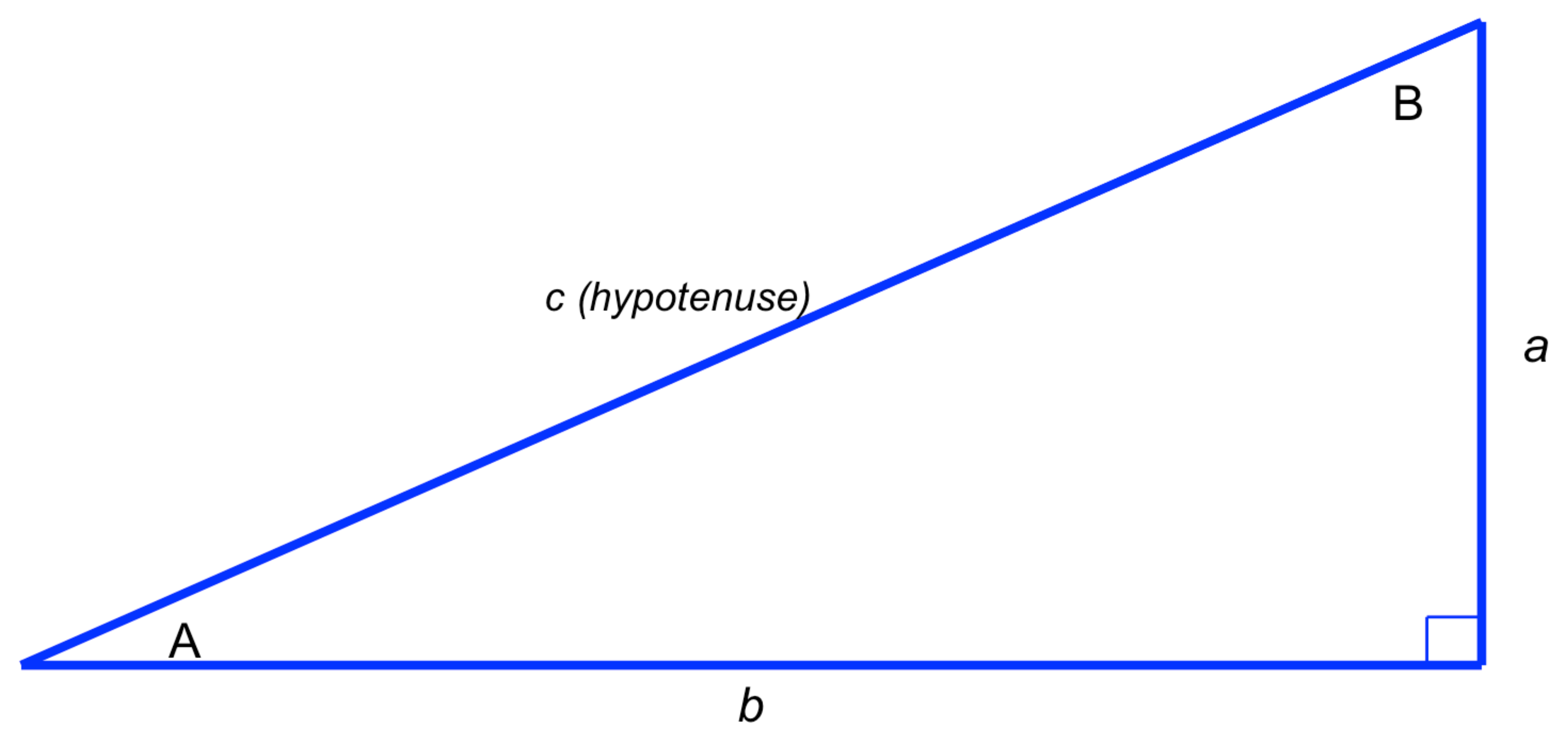
# Objectives

Today we close the loop. We return to the projectile problem from Day 23 — and now we have everything we need to finish it.

After today's class meeting, you should be able to:

- Apply the Pythagorean Theorem to find missing side lengths in right triangles.
- Use $\sin()$, $\cos()$, and $\tan()$ to relate angles and side lengths in right triangles.
- Use inverse trigonometric functions to find missing angles.
- Solve applied problems involving angles of elevation, depression, and right-triangle geometry.

# Right Triangle Labeling Convention



# The Pythagorean Theorem

**Theorem:** For any right triangle with legs $a$ and $b$ and hypotenuse $c$:

$$a^2 + b^2 = c^2$$

**Example 1:** Find the missing side length in each case.

(1) Legs $a = 5$ and $b = 7$. Find the hypotenuse $c$.

(2) Leg $a = 5$ and hypotenuse $c = 10$. Find the missing leg $b$.

**(1)**

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$$\begin{aligned} a^2 + b^2 &= c^2 \\ \implies 5^2 + 7^2 &= c^2 \\ \implies 25 + 49 &= c^2 \end{aligned}$$

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Note that we omit the "±" here since we know that $c$ is a *length*.

**(2)**

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$$\begin{aligned} a^2 + b^2 &= c^2 \\ \implies 5^2 + b^2 &= 10^2 \\ \implies 25 + b^2 &= 100 \end{aligned}$$

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$$\begin{aligned} a^2 + b^2 &= c^2 \\ \implies 5^2 + b^2 &= 10^2 \\ \implies 25 + b^2 &= 100 \\ \implies b^2 &= 75 \end{aligned}$$

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# Trig Ratios in Right Triangles

The unit circle definitions of **sin**, **cos**, and **tan** connect directly to side ratios in any right triangle. For angle $A$:

$$\sin(A) = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{a}{c} \qquad \cos(A) = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{b}{c} \qquad \tan(A) = \frac{\text{opposite}}{\text{adjacent}} = \frac{a}{b}$$

A useful mnemonic: **SOH-CAH-TOA**.

We can also find angles using the inverse functions:

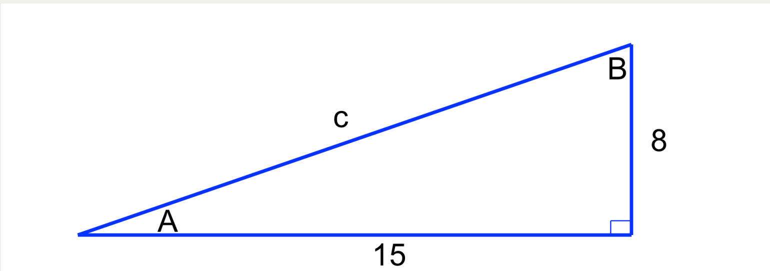
$$A = \arcsin\left(\frac{a}{c}\right) \qquad A = \arccos\left(\frac{b}{c}\right) \qquad A = \arctan\left(\frac{a}{b}\right)$$

**Returning to Non-Vertical Motion from Day 23**

On Day 23, we asked: if a ball is launched at $60^\circ$ with speed $64$ ft/sec, what is the maximum height? Now we can answer. The vertical component of velocity is $64\sin(60^\circ) = 64 \cdot \frac{\sqrt{3}}{2} = 32\sqrt{3}$ ft/sec. Substituting into $h\left(t\right) = -16t^2 + 32\sqrt{3}t$ gives a maximum height of 48 feet, reached at $t = \frac{\sqrt{3}}{1} \approx 1.73$ seconds.

# Solving Right Triangles I

**Example 2:** The right triangle below has $a = 8$ and $b = 15$. Find all missing sides and angles.




**Step 1: Find $c$ using the Pythagorean Theorem.**

$$8^2 + 15^2 = c^2$$

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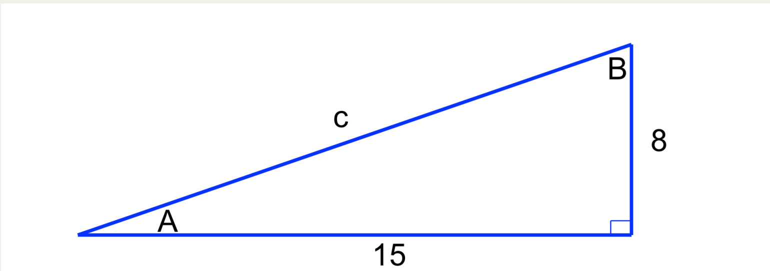


**Step 1: Find $c$ using the Pythagorean Theorem.**

$$8^2 + 15^2 = c^2$$
$$\implies 64 + 225 = c^2$$

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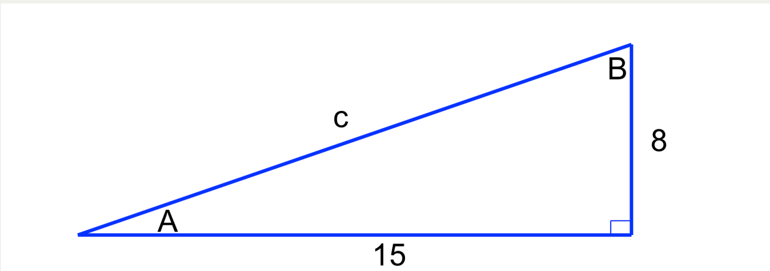


**Step 1: Find $c$ using the Pythagorean Theorem.**

$$\begin{aligned} 8^2 + 15^2 &= c^2 \\ \Longrightarrow 64 + 225 &= c^2 \\ \Longrightarrow 289 &= c^2 \\ \Longrightarrow c &= 17 \end{aligned}$$

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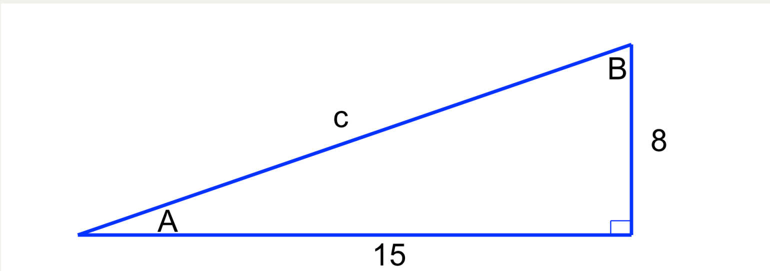


**Step 2: Find angle $A$ using $\sin(A) = \dfrac{a}{c}$.**

$$\sin(A) = \frac{8}{17}$$

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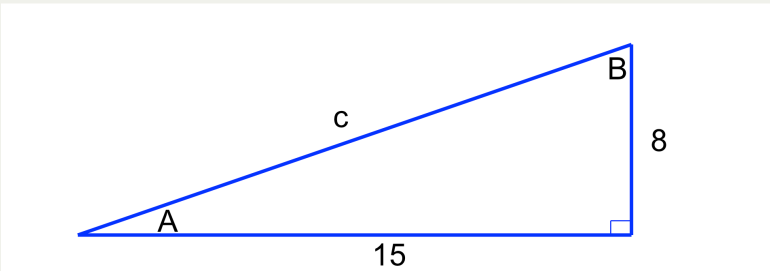


**Step 2: Find angle $A$ using $\sin(A) = \dfrac{a}{c}$.**

$$\sin(A) = \frac{8}{17}$$
$$\implies A = \arcsin\left(\frac{8}{17}\right)$$
$$\implies A \approx 28.07^\circ$$

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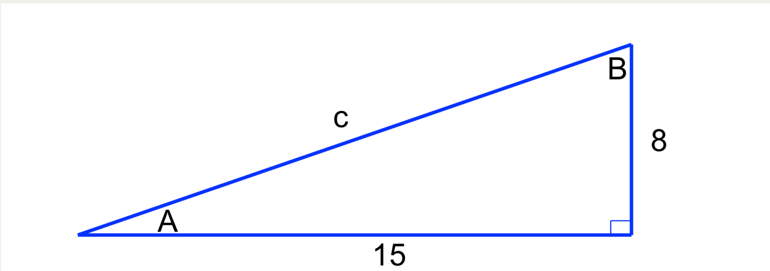


**Step 3: Find angle $B$ using the angle sum.**

$$A + B + 90^\circ = 180^\circ$$

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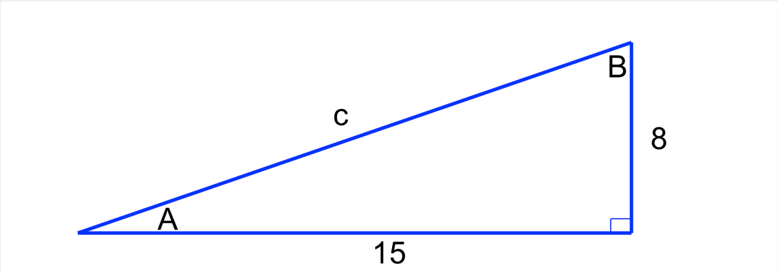



**Step 3: Find angle $B$ using the angle sum.**

$$\begin{aligned} A + B + 90^\circ &= 180^\circ \\ \implies 28.07^\circ + B + 90^\circ &\approx 180^\circ \\ \implies B &\approx 61.93^\circ \end{aligned}$$

The triangle is fully solved: $c = 17$, $A \approx 28.07^\circ$, $B \approx 61.93^\circ$.

# Triangle Practice

**Try It! 1:** Find all missing sides and angles for a right triangle with $A = 30^\circ$ and $a = 5$.

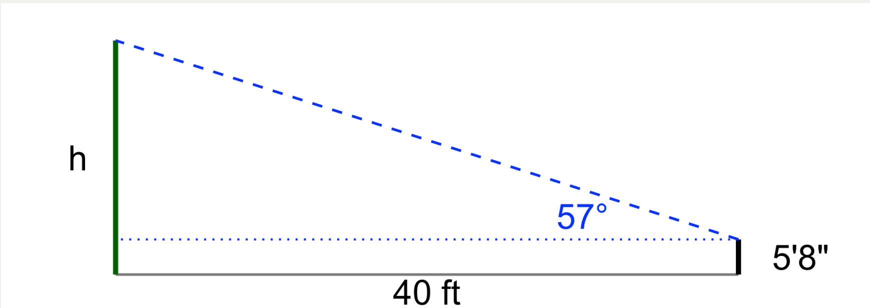
**Try It! 2:** Find all missing sides and angles for a right triangle with $B = 50^\circ$ and $c = 7$.

**Try It! 3:** Find all missing sides and angles for a right triangle with $A = 45^\circ$ and $c = 4$.

**Try It! 4:** Find all missing sides and angles for a right triangle with $a = 5$ and $b = 15$.

# Applied Example 1: Tree Height

**Problem:** An arborist who is $5'8''$ tall stands at the base of a tall tree and walks $40$ feet away. Using an inclinometer, they measure an angle of $57^\circ$ from eye level to the treetop. Estimate the height of the tree.




The height from eye level to the treetop uses $\tan$:

$$\tan(57^\circ) = \frac{h_{\text{eye}}}{40}$$

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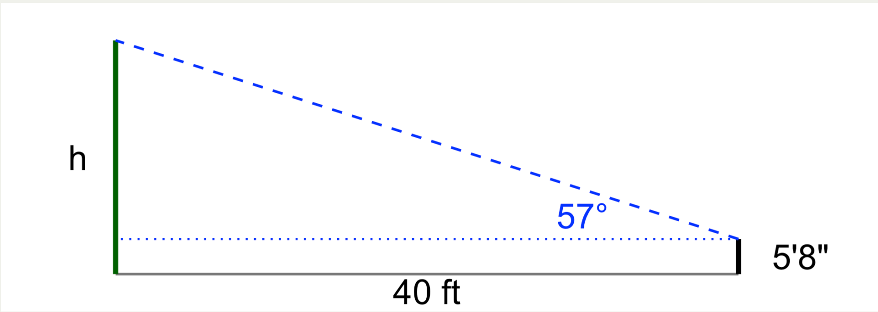



The height from eye level to the treetop uses **tan**:

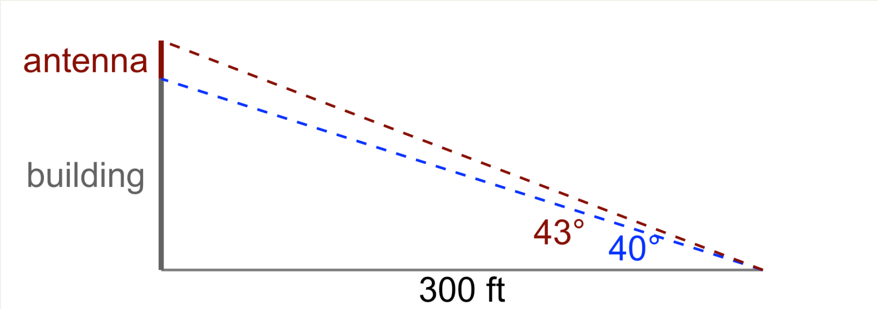
$$\tan(57^\circ) = \frac{h_{\text{eye}}}{40}$$
$$\implies h_{\text{eye}} = 40\tan(57^\circ)$$
$$\approx 61.59 \text{ ft}$$

Add the arborist's eye height ($5'8'' \approx 5.67$ ft):

$$h_{\text{tree}} \approx 61.59 + 5.67 \approx \boxed{67 \text{ ft}}$$

# Applied Example 2: Antenna Height

**Problem:** From a point $300$ ft from the base of a building, the angle of elevation to the top of the building is $40^\circ$ and to the top of a rooftop antenna is $43^\circ$. Find the height of the antenna.




Height of building top:

$$h_{\text{bldg}} = 300\tan(40^\circ) \approx 251.73 \text{ ft}$$

Height of antenna top:

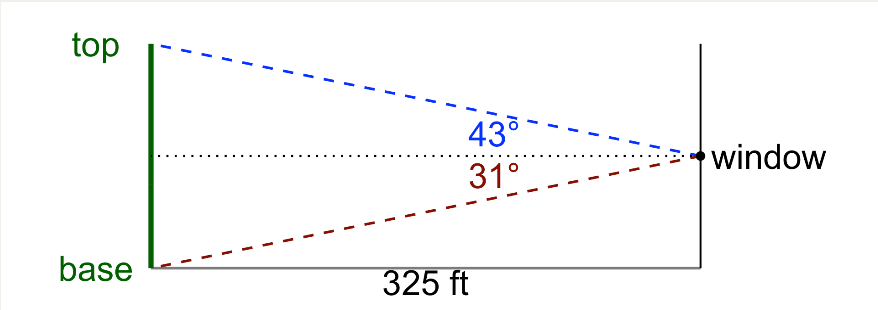
$$h_{\text{top}} = 300\tan(43^\circ) \approx 279.75 \text{ ft}$$

Antenna height:

$$h_{\text{antenna}} = 279.75 - 251.73 \approx \boxed{28 \text{ ft}}$$

# Applied Example 3: Radio Tower

**Problem:** A radio tower is $325$ ft from a building. From a window, the angle of elevation to the top of the tower is $43^\circ$ and the angle of depression to the base is $31^\circ$. How tall is the tower?




Height above the window:

$$h_{\text{above}} = 325\tan(43^\circ) \approx 303.07 \text{ ft}$$

Depth below the window:

$$h_{\text{below}} = 325\tan(31^\circ) \approx 195.28 \text{ ft}$$

Total tower height:

$$h_{\text{tower}} = 303.07 + 195.28 \approx \boxed{498 \text{ ft}}$$

# Exit Ticket Task

Navigate to our MAT142 Exit Ticket Form, answer the questions, and complete the *task* below.

*Note*. Today's discussion is listed as `21. Right Triangle Trigonometry`

**Task:** A surveyor standing at ground level measures the angle of elevation to the top of a cliff to be $38^\circ$. She then walks $50$ feet closer to the cliff and measures the angle of elevation to be $52^\circ$. How tall is the cliff?

# Summary and Congratulations

**Ideas From Today**

- In a right triangle, $\sin = \frac{\text{opp}}{\text{hyp}}$, $\cos = \frac{\text{adj}}{\text{hyp}}$, $\tan = \frac{\text{opp}}{\text{adj}}$ — **SOH-CAH-TOA**.
- Use $\arcsin()$, $\arccos()$, or $\arctan()$ to recover angles from side ratios.
- The angle sum of a triangle is $180^\circ$; once two angles are known, the third is determined.
- Angles of elevation and depression create right triangles whose sides correspond to horizontal distances and heights.

**Looking Ahead**

- This is the end of our content for MAT 142.
- The thread from Day 1 to today: we started with rates of change and limits, built up every class of algebraic function, then added a few completely new function classes (exponential, logarithmic, and trigonometric). Right triangle trig brings us back to where we started — ratios and geometry — but now with the full machinery of functions behind us.

**Thank you for a great semester.**

**Homework:**

Complete Homework 12 on MyOpenMath

Prepare for Exams