MAT 142: Radical Functions and Equations

Dr. Gilbert

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Reminders and Warm-Up

Last class we studied rational functions. Before moving on, let's practice solving a **rational equation** — and encounter a trap that will reappear today.

Problem 1: Solve $\dfrac{3}{x} + 2 = \dfrac{5}{x}$.

Hint. To start, multiply both sides by x (the simplest common denominator), then solve.

Problem 2: Solve $\dfrac{6}{x^2 - 9} + \dfrac{1}{x + 3} = \dfrac{2}{x - 3}$.

Hint. Start similarly and pay close attention to your solutions.

Extraneous Solutions

An *extraneous solution* is an algebraically valid result that fails when substituted back into the original equation — often because it violates a domain restriction. **Always check solutions** when multiplying by an expression containing a variable.

This same trap appears today with radical equations, where squaring both sides can introduce extraneous solutions.

Objectives

Today we study **radical functions** — functions built from roots of expressions.

After today's class meeting, you should be able to:

- Identify the domain of a radical function based on the index of the root.
- Solve polynomial inequalities using sign testing.
- Find the roots and y-intercept of a radical function.
- Sketch the graph of a radical function.

What Is a Radical Function?

Definition (Radical Function): A function of the form

$$f(x) = \sqrt[n]{g(x)} = (g(x))^{1/n}$$

where n is a positive integer, is called a *radical* (or *root*) function.

Domain depends on the index n:

- If n is **odd**: the domain is all real numbers (odd roots of negatives are defined).
- If n is **even**: the domain requires $g(x) \geq 0$ (even roots of negatives result in complex numbers).

Roots: $f(x) = 0$ only when $g(x) = 0$.

***y*-intercept:** Evaluate $f(0)$, provided 0 is in the domain.

> **On Notation**
>
> While the root notation ($\sqrt{}$) is common, it is almost always more mathematically convenient to use the exponent notation.
>
> For example, instead of witing $f(x) = \sqrt{x}$, write $f(x) = x^{1/2}$

Domain of Radical Functions

Example 1: Find the domain of the radical function $f(x) = 3\sqrt{(x-9)} + 17$

Solution.

Since the order of the root is even, we need the argument (the expression inside of the square root) to be non-negative.

$$x - 9 \geq 0$$

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$$\begin{aligned} x-9 &\geq 0 \\ \Longrightarrow \quad x &\geq 9 \end{aligned}$$

The domain of $f\left(x\right)$ is $x\geq 9$, or equivalently $[9,\infty)$

Example 2: Find the domain of the radical function $g\left(x\right)=5\left(\dfrac{x+8}{7}\right)^{1/7}-4$.

Solution.

Since the order of the root is odd (it is a *seventh* root), the domain consists of *all real numbers*, or equivalently $(-\infty,\infty)$.

Domain of Radical Functions Practice

Before encountering a comprehensive example, let's build intuition for domains. Find the domain of each function:

Try It! 1: Find the domain of the function $f\left(x\right) = \sqrt{\left(\frac{3x + 4}{8}\right)}$.

Try It! 2: Find the domain of the function $g\left(x\right) = 3\sqrt[5]{\left(x^2 - 9x + 20\right)}$.

Try It! 3: Find the domain of the function $h\left(x\right) = -2\left(x^2 - 9x + 20\right)^{1/4} + 10$.

Try It! 4: Find the domain of the function $j\left(x\right) = \frac{5}{\sqrt{\left(8 - 2x\right)}}$

There are several interesting things to consider across the examples above.

Solving Polynomial Inequalities

Finding domains of even-degree radical functions requires solving inequalities like $g(x) \geq 0$. In all but the simplest cases (like **Example 1** and **Try It! 1**), we use *sign testing*.

Strategy (Sign Testing): To find the domain of a function that includes $(g(x))^{\frac{1}{2n}}$, follow the steps below.

1. Find the roots of $g(x)$ — these are the potential boundary locations.
2. The roots divide the number line into intervals.
3. Test one point in each interval to determine the sign of $g(x)$ there.
4. The domain for this part of the function consists of intervals where the sign is positive (or zero, if it is permissible for $(g(x))^{\frac{1}{2n}}$ to evaluate to 0).

The function can only change sign at its roots, so one test point per interval is sufficient.

More Domains of Radical Functions

Example 3: Find the domain of the radical function $f(x) = \left(x^2 - 5x - 14\right)^{1/8}$

Solution.

Since the root is even order (8th root), we need to consider domain restrictions.

The function $f(x)$ will only be defined when $x^2 - 5x - 14 \geq 0$

We'll start by finding the boundary values where $x^2 - 5x - 14 = 0$.

$$x^2 - 5x - 14 = 0$$

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$$\begin{aligned} x^2 - 5x - 14 &= 0 \\ \implies (x-7)(x+2) &= 0 \end{aligned}$$

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The analysis above results in three sub-intervals where the expression under the root is not zero: $(-\infty, -2)$, $(-2, 7)$, and $(7, \infty)$.

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Conduct Sign Analysis:

Can either evaluate the sign (positive/negative) from $x^2 - 5x - 14$ or $(x-7)(x+2)$.

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Solution.

Again, we need the expression under the root to be non-negative.

Notice that the boundary values (the x-values resulting in 0 under the radical) are $x=-5$, -1, 2, and 6

We'll draw a number line and conduct the sign analysis.

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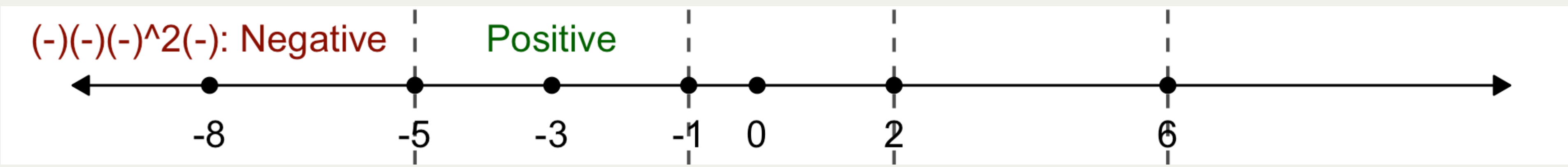

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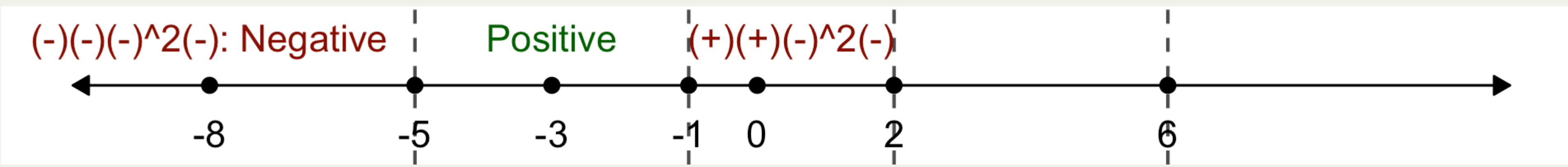

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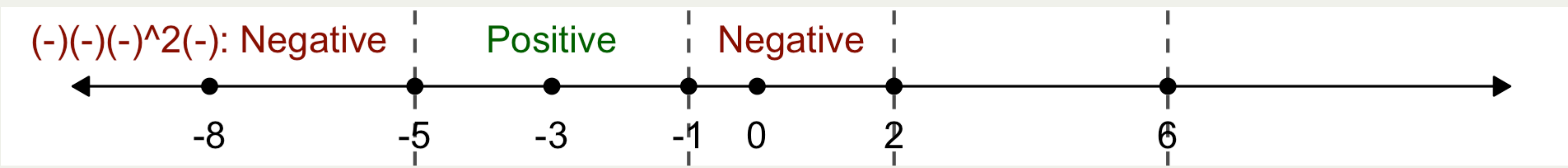

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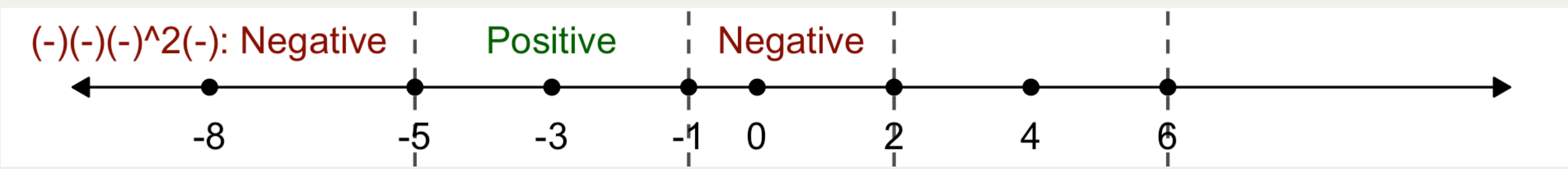

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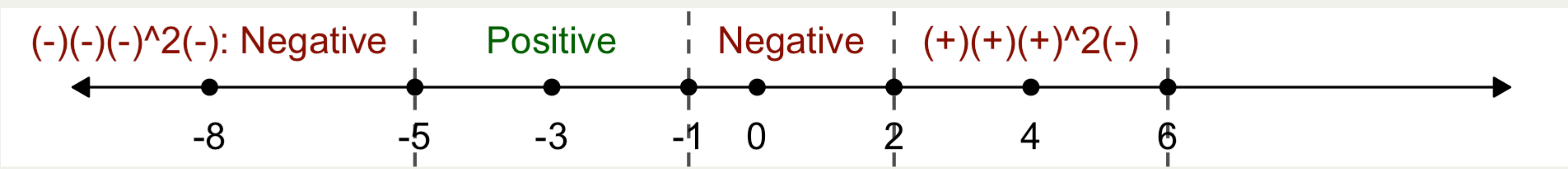

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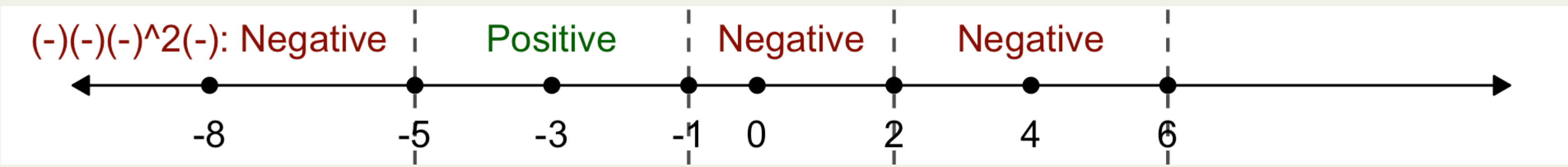

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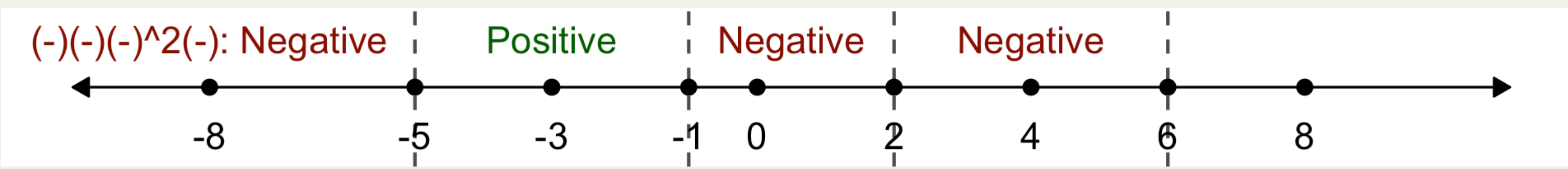

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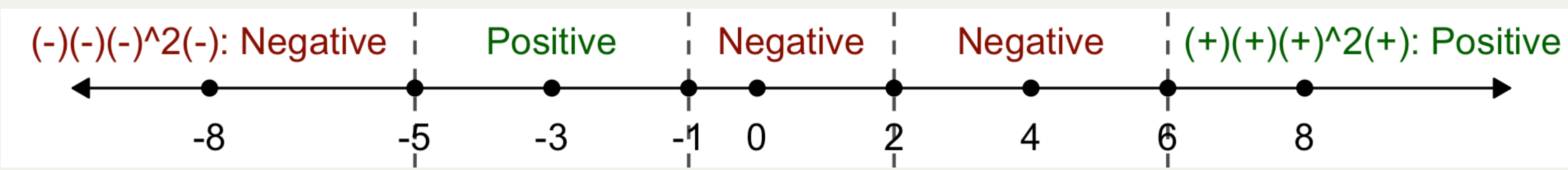

All of the boundary values and the segments corresponding to positive values under the radical are included in the domain.

We'll use $\{2\}$ to indicate the single value 2 when we write the domain.

Domain: $\boxed{[-5,-1] \cup \{2\} \cup [6,\infty)}$

Analysis of Radical Functions I

Example 5: Find the domain, roots, y-intercept, and end behavior of $f(x) = \sqrt{(x^2 - 11x - 26)}$. Use your results to sketch the graph of $f(x)$

Solution.

Since the index is 2 (even), we need $x^2 - 11x - 26 \geq 0$.

Step 1: Find roots of $x^2 - 11x - 26 = 0$.

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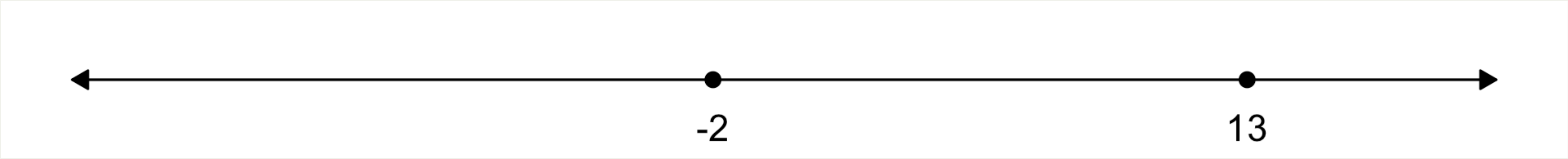
Step 2: These roots create three intervals

$(-\infty, -2)$, $(-2, 13)$, and $(13, \infty)$



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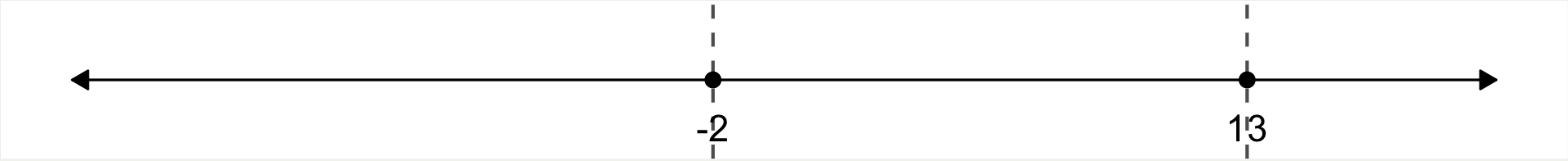

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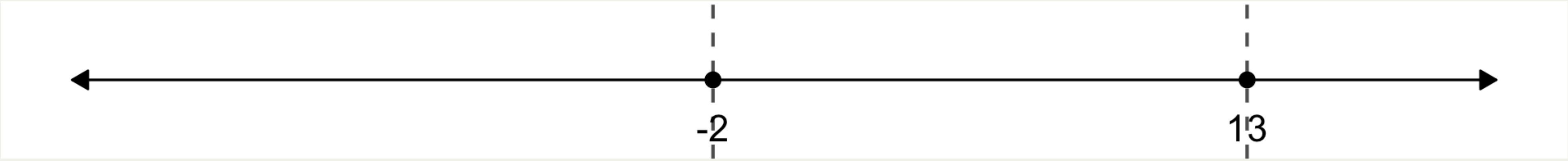

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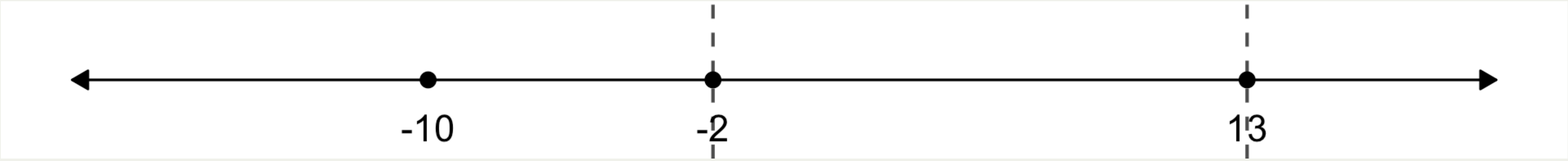

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$g(-10) = (-10)^2 - 11(-10) - 26$

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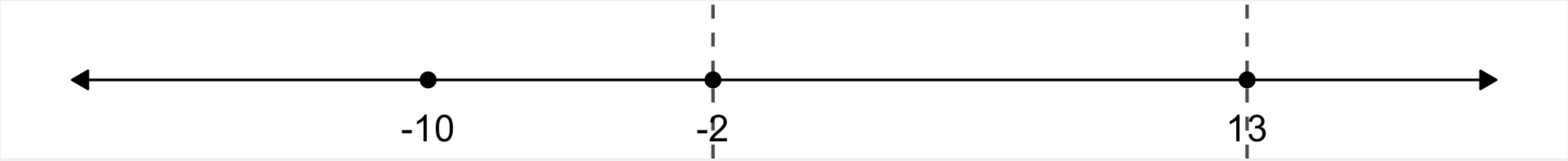
$$\begin{aligned} x^2 - 11x - 26 &= 0 \\ \implies (x-13)(x+2) &= 0 \\ \implies x = 13 \text{ or } x &= -2 \end{aligned}$$

Step 2: These roots create three intervals

$(-\infty, -2)$, $(-2, 13)$, and $(13, \infty)$



Step 3: Test one point in each interval.

$g(-10) = (100 + 110 - 26)$

Analysis of Radical Functions I

Example 5: Find the domain, roots, y-intercept, and end behavior of $f(x) = \sqrt{(x^2 - 11x - 26)}$. Use your results to sketch the graph of $f(x)$

Solution.

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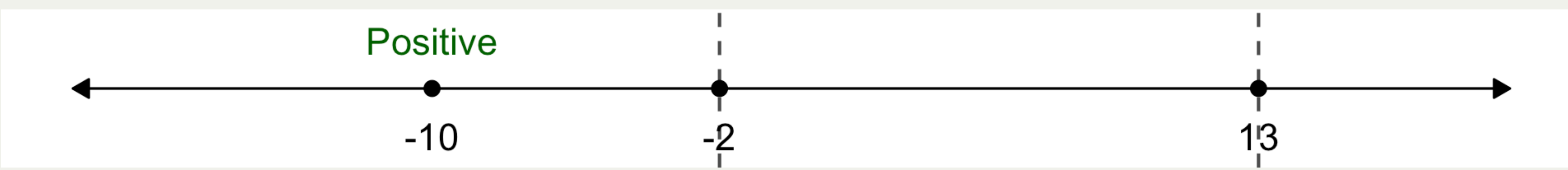
Step 3: Test one point in each interval.

$g(-10) = 184$



Analysis of Radical Functions I

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Solution.

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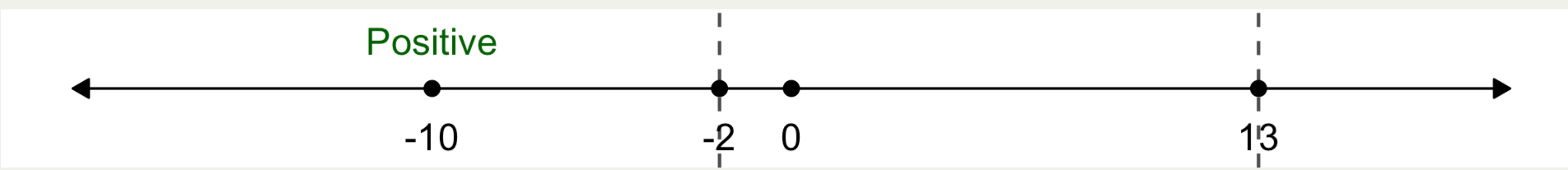
Step 1: Find roots of $x^2 - 11x - 26 = 0$.

$$x^2 - 11x - 26 = 0$$
$$\implies (x - 13)(x + 2) = 0$$
$$\implies x = 13 \text{ or } x = -2$$

Step 2: These roots create three intervals

$(-\infty, -2)$, $(-2, 13)$, and $(13, \infty)$



Step 3: Test one point in each interval.

$g(-10) = 184$

Analysis of Radical Functions I

Example 5: Find the domain, roots, y-intercept, and end behavior of $f(x) = \sqrt{(x^2 - 11x - 26)}$. Use your results to sketch the graph of $f(x)$

Solution.

Since the index is 2 (even), we need $x^2 - 11x - 26 \geq 0$.

Step 1: Find roots of $x^2 - 11x - 26 = 0$.

$$\begin{aligned} x^2 - 11x - 26 &= 0 \\ \implies (x-13)(x+2) &= 0 \\ \implies x = 13 \text{ or } x &= -2 \end{aligned}$$

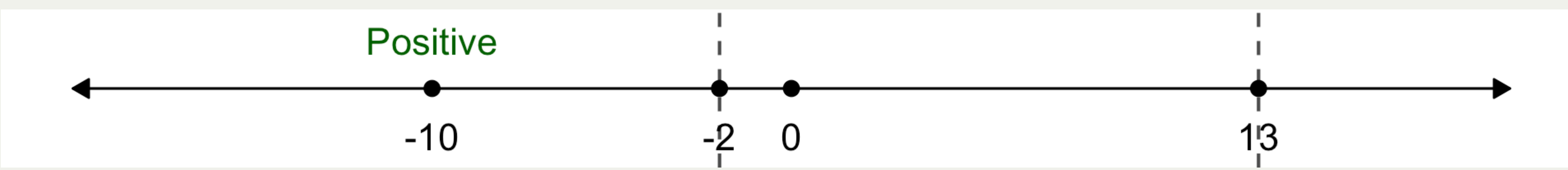
Step 2: These roots create three intervals

$(-\infty, -2)$, $(-2, 13)$, and $(13, \infty)$

Step 3: Test one point in each interval.

$g(-10) = 184$

$g(0) = (0)^2 - 11(0) - 26$



Analysis of Radical Functions I

Example 5: Find the domain, roots, y-intercept, and end behavior of $f(x) = \sqrt{(x^2 - 11x - 26)}$. Use your results to sketch the graph of $f(x)$

Solution.

Since the index is 2 (even), we need $x^2 - 11x - 26 \geq 0$.

Step 1: Find roots of $x^2 - 11x - 26 = 0$.

$$\begin{aligned} x^2 - 11x - 26 &= 0 \\ \implies (x-13)(x+2) &= 0 \\ \implies x = 13 \text{ or } x &= -2 \end{aligned}$$

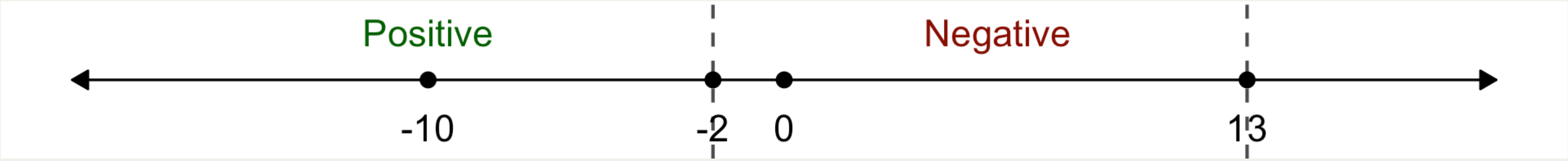
Step 2: These roots create three intervals

$(-\infty, -2)$, $(-2, 13)$, and $(13, \infty)$

Step 3: Test one point in each interval.

$g(-10) = 184$

$g(0) = -26$



Analysis of Radical Functions I

Example 5: Find the domain, roots, y-intercept, and end behavior of $f(x) = \sqrt{(x^2 - 11x - 26)}$. Use your results to sketch the graph of $f(x)$

Solution.

Since the index is 2 (even), we need $x^2 - 11x - 26 \geq 0$.

Step 1: Find roots of $x^2 - 11x - 26 = 0$.

$$\begin{aligned} x^2 - 11x - 26 &= 0 \\ \implies (x-13)(x+2) &= 0 \\ \implies x = 13 \text{ or } x &= -2 \end{aligned}$$

Step 2: These roots create three intervals

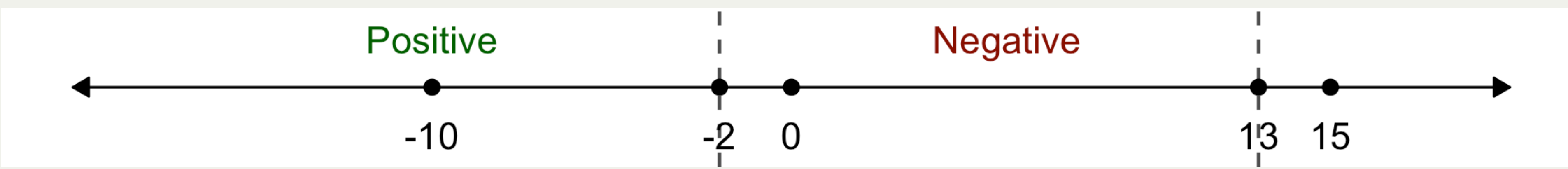
$(-\infty, -2)$, $(-2, 13)$, and $(13, \infty)$

Step 3: Test one point in each interval.

$g(-10) = 184$

$g(0) = -26$

$g(15) = (15)^2 - 11(15) - 26$



Analysis of Radical Functions I

Example 5: Find the domain, roots, y-intercept, and end behavior of $f(x) = \sqrt{(x^2 - 11x - 26)}$. Use your results to sketch the graph of $f(x)$

Solution.

Since the index is 2 (even), we need $x^2 - 11x - 26 \geq 0$.

Step 1: Find roots of $x^2 - 11x - 26 = 0$.

$$\begin{aligned} x^2 - 11x - 26 &= 0 \\ \implies (x-13)(x+2) &= 0 \\ \implies x = 13 \text{ or } x &= -2 \end{aligned}$$

Step 2: These roots create three intervals

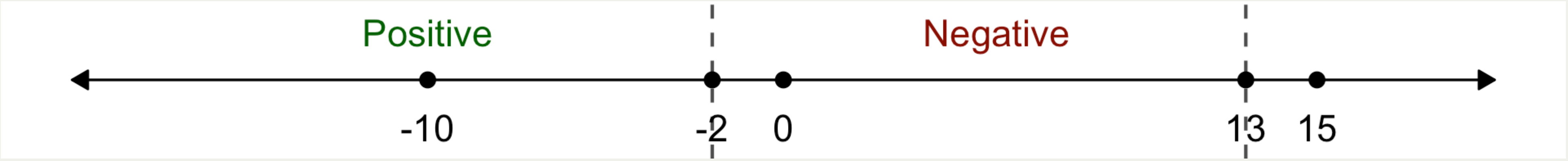
$(-\infty, -2)$, $(-2, 13)$, and $(13, \infty)$

Step 3: Test one point in each interval.

$g(-10) = 184$

$g(0) = -26$

$g(15) = 225 - 165 - 26$



Analysis of Radical Functions I

Example 5: Find the domain, roots, y-intercept, and end behavior of $f(x) = \sqrt{(x^2 - 11x - 26)}$. Use your results to sketch the graph of $f(x)$

Solution.

Since the index is 2 (even), we need $x^2 - 11x - 26 \geq 0$.

Step 1: Find roots of $x^2 - 11x - 26 = 0$.

$$\begin{aligned} & x^2 - 11x - 26 = 0 \\ \implies & (x - 13)(x + 2) = 0 \\ & \implies x = 13 \text{ or } x = -2 \end{aligned}$$

Step 2: These roots create three intervals

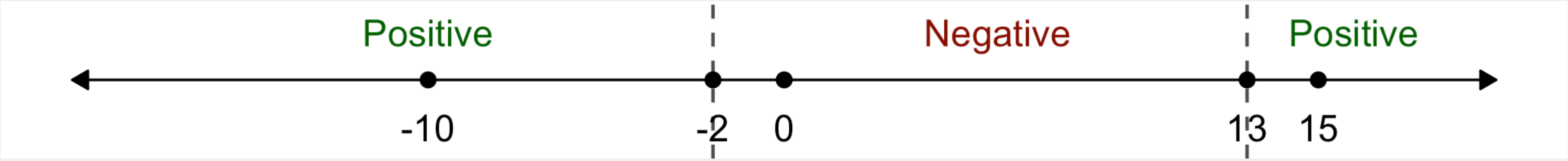
$(-\infty, -2)$, $(-2, 13)$, and $(13, \infty)$

Step 3: Test one point in each interval.

$g(-10) = 184$

$g(0) = -26$

$g(15) = 34$



Analysis of Radical Functions I

Example 5: Find the domain, roots, y-intercept, and end behavior of $f(x) = \sqrt{(x^2 - 11x - 26)}$. Use your results to sketch the graph of $f(x)$

Solution.

Since the index is 2 (even), we need $x^2 - 11x - 26 \geq 0$.

Step 1: Find roots of $x^2 - 11x - 26 = 0$.

$$\begin{aligned} x^2 - 11x - 26 &= 0 \\ \implies (x-13)(x+2) &= 0 \\ \implies x &= 13 \text{ or } x = -2 \end{aligned}$$

Step 2: These roots create three intervals

$(-\infty, -2)$, $(-2, 13)$, and $(13, \infty)$

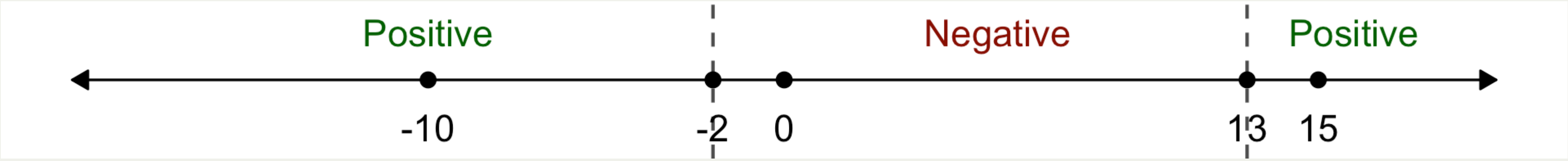
Step 3: Test one point in each interval.

$g(-10) = 184$

$g(0) = -26$

$g(15) = 34$

Domain: $\boxed{(-\infty, -2] \cup [13, \infty)}$



Analysis of Radical Functions I

Example 5: Find the domain, roots, y-intercept, and end behavior of $f(x) = \sqrt{\left(x^2 - 11x - 26\right)}$. Use your results to sketch the graph of $f(x)$

Solution.

Domain: $\boxed{(-\infty, -2] \cup [13, \infty)}$

Roots: We'll solve $f(x) = 0$

$$\sqrt{\left(x^2 - 11x - 26\right)} = 0$$

Analysis of Radical Functions I

Example 5: Find the domain, roots, y-intercept, and end behavior of $f(x) = \sqrt{(x^2 - 11x - 26)}$. Use your results to sketch the graph of $f(x)$

Solution.

Domain: $\boxed{(-\infty, -2] \cup [13, \infty)}$

Roots: We'll solve $f(x) = 0$

$$\sqrt{(x^2 - 11x - 26)} = 0$$
$$\implies x^2 - 11x - 26 = 0$$

Analysis of Radical Functions I

Example 5: Find the domain, roots, y-intercept, and end behavior of $f(x) = \sqrt{(x^2 - 11x - 26)}$. Use your results to sketch the graph of $f(x)$

Solution.

Domain: $\boxed{(-\infty, -2] \cup [13, \infty)}$

Roots: We'll solve $f(x) = 0$

$$\begin{aligned} \sqrt{(x^2 - 11x - 26)} &= 0 \\ \implies x^2 - 11x - 26 &= 0 \\ \implies (x - 13)(x + 2) &= 0 \end{aligned}$$

Analysis of Radical Functions I

Example 5: Find the domain, roots, y-intercept, and end behavior of $f(x) = \sqrt{(x^2 - 11x - 26)}$. Use your results to sketch the graph of $f(x)$

Solution.

Domain: $\boxed{(-\infty, -2] \cup [13, \infty)}$

Roots: We'll solve $f(x) = 0$

$$\sqrt{(x^2 - 11x - 26)} = 0$$
$$\implies x^2 - 11x - 26 = 0$$
$$\implies (x - 13)(x + 2) = 0$$
$$\implies x = -2 \text{ or } x = 13$$

Analysis of Radical Functions I

Example 5: Find the domain, roots, y-intercept, and end behavior of $f(x) = \sqrt{(x^2 - 11x - 26)}$. Use your results to sketch the graph of $f(x)$

Solution.

Domain: $\boxed{(-\infty, -2] \cup [13, \infty)}$

Roots: The roots of $f(x)$ are at $\boxed{(-2, 0)}$ and $\boxed{(13, 0)}$

y-intercept: Note that $x = 0$ is not in the domain of the function. This means that there is no y-intercept.

End behavior: We'll analyse $\lim\limits_{x\to-\infty} f(x)$ and $\lim\limits_{x\to\infty} f(x)$

$$\lim_{x\to-\infty} f(x) = \lim_{x\to-\infty} \sqrt{(x^2 - 11x - 26)}$$

The quadratic under the root goes off to infinity.

Because of this, the root goes off to infinity as well.

$$\lim_{x\to-\infty} f(x) = \infty$$

$$\lim_{x\to\infty} f(x) = \lim_{x\to\infty} \sqrt{(x^2 - 11x - 26)}$$

The quadratic under the root goes off to infinity.

Because of this, the root goes off to infinity as well.

$$\lim_{x\to\infty} f(x) = \infty$$

Analysis of Radical Functions I

Example 5: Find the domain, roots, y-intercept, and end behavior of $f(x) = \sqrt{(x^2 - 11x - 26)}$. Use your results to sketch the graph of $f(x)$

Solution.

Domain: $\boxed{(-\infty, -2] \cup [13, \infty)}$

Roots: The roots of $f(x)$ are at $\boxed{(-2, 0)}$ and $\boxed{(13, 0)}$

y-intercept: Note that $x = 0$ is not in the domain of the function. This means that there is no y-intercept.

End behavior: We've found $\lim_{x \to -\infty} f(x) = \infty$ and $\lim_{x \to \infty} f(x) = \infty$

Analysis of Radical Functions II

Example 6: Find the domain, roots, y-intercept, and end behavior of $g(x) = \sqrt{(16 - x^2)}$. Use your results to sketch the graph of $g(x)$.

Solution.

Domain: We'll start by finding the boundary values.

$$16 - x^2 = 0$$

Analysis of Radical Functions II

Example 6: Find the domain, roots, y-intercept, and end behavior of $g\left(x\right) = \sqrt{\left(16 - x^2\right)}$. Use your results to sketch the graph of $g\left(x\right)$.

Solution.

Domain: We'll start by finding the boundary values.

$$\begin{aligned} 16 - x^2 &= 0 \\ \implies x^2 &= 16 \end{aligned}$$

Analysis of Radical Functions II

Example 6: Find the domain, roots, y-intercept, and end behavior of $g(x) = \sqrt{(16 - x^2)}$. Use your results to sketch the graph of $g(x)$.

Solution.

Domain: We'll start by finding the boundary values.

$$\begin{aligned} 16 - x^2 &= 0 \\ \implies x^2 &= 16 \\ \implies x &= \pm 4 \end{aligned}$$

Analysis of Radical Functions II

Example 6: Find the domain, roots, y-intercept, and end behavior of $g(x) = \sqrt{(16 - x^2)}$. Use your results to sketch the graph of $g(x)$.

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Analysis of Radical Functions II

Example 6: Find the domain, roots, y-intercept, and end behavior of $g\left(x\right) = \sqrt{\left(16 - x^2\right)}$. Use your results to sketch the graph of $g\left(x\right)$.

Solution.

Domain: We'll start by finding the boundary values.

$$\begin{aligned} 16 - x^2 &= 0 \\ \implies x^2 &= 16 \\ \implies x &= \pm 4 \end{aligned}$$

Analysis of Radical Functions II

Example 6: Find the domain, roots, y-intercept, and end behavior of $g\left(x\right)=\sqrt{\left(16-x^2\right)}$. Use your results to sketch the graph of $g\left(x\right)$.

Solution.

Domain: We'll start by finding the boundary values.

$$16-x^2=0$$
$$\implies x^2=16$$
$$\implies x=\pm 4$$

Analysis of Radical Functions II

Example 6: Find the domain, roots, y-intercept, and end behavior of $g\left(x\right) = \sqrt{\left(16 - x^2\right)}$. Use your results to sketch the graph of $g\left(x\right)$.

Solution.

Domain: $[-4, 4]$

Roots: We'll solve $g\left(x\right) = 0$

$$\sqrt{\left(16 - x^2\right)} = 0$$

Analysis of Radical Functions II

Example 6: Find the domain, roots, y-intercept, and end behavior of $g\left(x\right) = \sqrt{\left(16 - x^2\right)}$. Use your results to sketch the graph of $g\left(x\right)$.

Solution.

Domain: $[-4, 4]$

Roots: We'll solve $g\left(x\right) = 0$

$$\begin{aligned} \sqrt{\left(16 - x^2\right)} &= 0 \\ \implies 16 - x^2 &= 0 \end{aligned}$$

Analysis of Radical Functions II

Example 6: Find the domain, roots, y-intercept, and end behavior of $g(x) = \sqrt{(16 - x^2)}$. Use your results to sketch the graph of $g(x)$.

Solution.

Domain: $[-4, 4]$

Roots: We'll solve $g(x) = 0$

$$\begin{aligned} \sqrt{(16 - x^2)} &= 0 \\ \implies 16 - x^2 &= 0 \\ \implies x^2 &= 16 \end{aligned}$$

Analysis of Radical Functions II

Example 6: Find the domain, roots, y-intercept, and end behavior of $g\left(x\right) = \sqrt{\left(16 - x^2\right)}$. Use your results to sketch the graph of $g\left(x\right)$.

Solution.

Domain: $[-4, 4]$

Roots: We'll solve $g\left(x\right) = 0$

$$\begin{aligned} \sqrt{\left(16 - x^2\right)} &= 0 \\ \implies 16 - x^2 &= 0 \\ \implies x^2 &= 16 \\ \implies x &= \pm 4 \end{aligned}$$

There's no surprise here. The roots are the boundary values we found earlier.

Analysis of Radical Functions II

Example 6: Find the domain, roots, y-intercept, and end behavior of $g\left(x\right)=\sqrt{\left(16-x^2\right)}$. Use your results to sketch the graph of $g\left(x\right)$.

Solution.

Domain: $[-4,4]$

Roots: The roots of $g\left(x\right)$ are at $(-4,0)$ and $(4,0)$.

y-intercept: We'll evaluate $g\left(0\right)$.

$$g\left(0\right)=\sqrt{\left(16-\left(0\right)^2\right)}$$

Analysis of Radical Functions II

Example 6: Find the domain, roots, y-intercept, and end behavior of $g(x) = \sqrt{(16 - x^2)}$. Use your results to sketch the graph of $g(x)$.

Solution.

Domain: $[-4, 4]$

Roots: The roots of $g(x)$ are at $(-4, 0)$ and $(4, 0)$.

y-intercept: We'll evaluate $g(0)$.

$$\begin{aligned} g(0) &= \sqrt{\left(16 - (0)^2\right)} \\ &= \sqrt{16} \end{aligned}$$

Analysis of Radical Functions II

Example 6: Find the domain, roots, y-intercept, and end behavior of $g(x) = \sqrt{\left(16 - x^2\right)}$. Use your results to sketch the graph of $g(x)$.

Solution.

Domain: $[-4, 4]$

Roots: The roots of $g(x)$ are at $(-4, 0)$ and $(4, 0)$.

y-intercept: We'll evaluate $g(0)$.

$$\begin{aligned} g(0) &= \sqrt{\left(16 - (0)^2\right)} \\ &= \sqrt{16} \\ &= 4 \end{aligned}$$

Analysis of Radical Functions II

Example 6: Find the domain, roots, y-intercept, and end behavior of $g(x) = \sqrt{(16 - x^2)}$. Use your results to sketch the graph of $g(x)$.

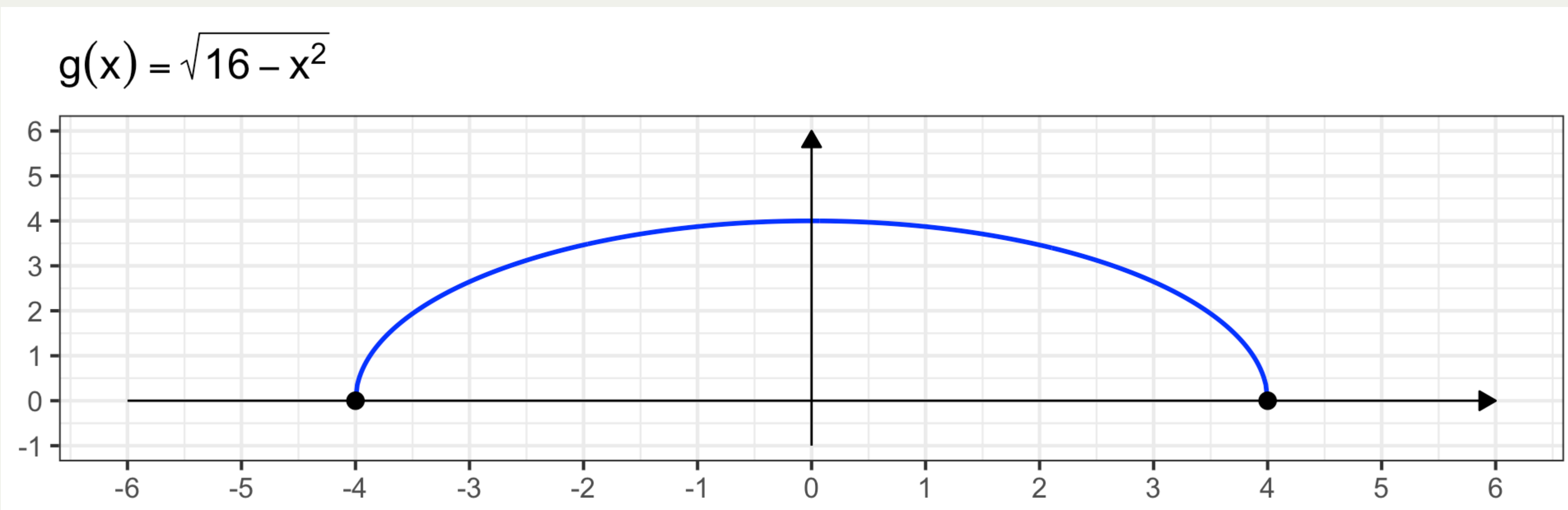
Solution.

Domain: $[-4, 4]$

Roots: The roots of $g(x)$ are at $(-4, 0)$ and $(4, 0)$.

y-intercept: The y-intercept of the function is at $(0, 4)$.

End Behavior: Not applicable since the domain does not extend toward positive or negative infinity.



Radical Function Practice

For each radical function below, find: (i) the domain, (ii) any roots, and (iii) the y-intercept (if one exists).

Try It! 1: $f\left(x\right)=\sqrt{\left(4-2x\right)}$

Try It! 2: $g\left(x\right)=\sqrt{\left(3x^2-4x-15\right)}$

Try It! 3: $h\left(x\right)=\sqrt{\left(x^2-9\right)}$

Try It! 4: $j\left(x\right)=\sqrt{\left(x^2+9\right)}$

Try It! 5: $k\left(x\right)=\sqrt{\left(x+3\right)^2}$

> **Challenge for Try It! 5:** Before computing, predict what the graph will look like. Then simplify $\sqrt{\left(x+3\right)^2}$ algebraically. Does the result match your prediction?

Radical Functions and Inverse Functions

Radical functions arise naturally as *inverses of power functions*.

For example, $f(x) = x^2$ is not invertible over all of $\mathbb{R}$ — it fails the horizontal line test. But if we restrict its domain to $x \geq 0$, then $f(x) = x^2$ is invertible, and its inverse is $f^{-1}(x) = \sqrt{x}$.

This is one justification for why $\sqrt{x}$ is only defined for $x \geq 0$. It's the inverse of a function whose restricted domain was $x \geq 0$.

More generally, if n is even, then $f(x) = x^n$ restricted to $x \geq 0$ has inverse $f^{-1}(x) = x^{1/n} = \sqrt[n]{x}$.

For *odd* n, no domain restriction is needed because $f(x) = x^n$ already passes the horizontal line test on all of $\mathbb{R}$.

Exit Ticket Task

Navigate to our MAT142 Exit Ticket Form, answer the questions, and complete the *task* below.

Note. Today's discussion is listed as `15. Radical Functions and Equations`

Task: Consider $f(x) = \sqrt{(x^2 - 4x - 12)}$.

(a) Find the domain of $f(x)$ using sign testing.

(b) Find any roots and the y-intercept (if one exists).

(c) Sketch a rough graph.

Summary and Next Time...

Ideas From Today

- A **radical function** $f(x) = (g(x))^{1/n}$ has domain that depends on whether n is even or odd.
  - Odd index: all real numbers.
  - Even index: solve $g(x) \geq 0$ using sign testing.
- **Sign testing:** find roots of g, test one point per interval, keep intervals where the sign is non-negative.
- **Roots** of f occur where $g(x) = 0$.
- Radical functions arise as inverses of power functions with domain restrictions.
- **Extraneous solutions** can appear whenever you multiply both sides by a variable expression (rational equations) or raise both sides to a power (radical equations) — always check solutions.

Looking Ahead

- With algebraic functions complete, we move into **exponential and logarithmic functions**.
- These are genuinely new classes of function. For example, exponential functions have the independent variable in the exponent.
- The inverse relationship between exponentials and logarithms connects directly back to our Day 9 discussion on inverse functions.
  - Take a look back at that discussion for a refresher.

Next Time:
Basics of Exponentials and Logarithms
Homework: