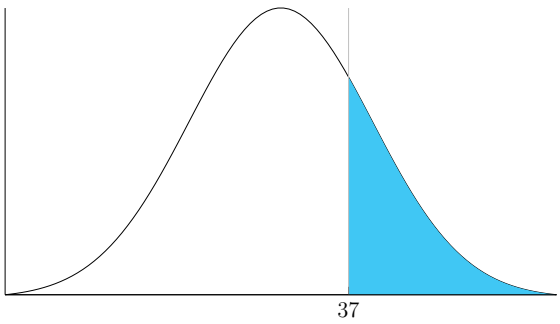
Example 1 *The distribution of the number of eggs laid by a certain species of hen during their breeding period is normally distributed with a mean of 35 eggs with a standard deviation of 18.2 eggs. Suppose a group of researchers randomly samples 45 hens of this species, counts the number of eggs laid during their breeding period, and records a sample mean of 37 eggs. Find the probability of observing a sample of 45 hens whose mean number of eggs laid during the breeding period is at least 37.*

Solution. Notice that the last sentence of the problem asks us to find the probability of an event. We know that `R` has the `pnorm` and `pt` functions to help us find probabilities. We can use `pnorm` when we know σ (the population standard deviation) or if we are working with proportions, but must use `pt` otherwise. The `pnorm` function in `R` is super nice because we don't need to first convert to a distribution centered at 0 and with standard deviation 1. We can execute: `pnorm(boundary, mean, SE)` to find the area underneath the normal curve and to the left of the boundary value.

Notice that we are working with the mean of a single-sample of 45 hens. Since this is the case we can use the Standard Error Decision Tree to find that $S_E = \sigma/\sqrt{n}$. That is, our standard error is $S_E = 18.2/\sqrt{45} \approx 2.71$. Now, we use `pnorm` to find our probability:



$$\begin{aligned} P\left[x \geq 37\right] &= 1 - pnorm(37, 35, 2.71) \\ &\approx 1 - .7697 \\ &= .2303 \end{aligned}$$

That is, the probability of observing a random sample of 45 of these hens who lay an average of at least 37 eggs during their breeding period is about 23%.