

Example 1 (Hypothesis Test) *A recent study looked at average ages of male and female inmates on Death Row. The random sample of 9 women and 14 men on death row resulted in the following:*

	Men	Women
$\bar{x}$	39	44
s	4.5	6.2

Conduct a hypothesis test to determine whether there is strong evidence to suggest that the men on Death Row are, on average, younger than women on Death Row. Use $\alpha = 0.05$.

Solution. Notice that we are asked to conduct a hypothesis test – keep in mind that, even if the prompt didn’t request a hypothesis test explicitly, any request for “evidence” leads us to conduct a hypothesis test.

Step 1: Hypotheses:

$$\begin{aligned} H_0 : \mu_{\text{women}} &= \mu_{\text{men}} \\ H_a : \mu_{\text{men}} &< \mu_{\text{women}} \end{aligned}$$

Step 2: Set level of significance

Let $\alpha = 0.05$ means that unless observing a sample like ours (at least as favorable to H_a) is less than 5% likely, then we cannot reject the null hypothesis (H_0).

Step 3: Compute test statistic and p -value.

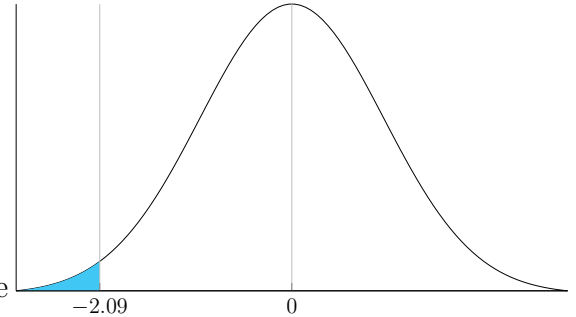
Notice that we are conducting a hypothesis test involving means (μ ’s) – we have two samples – we don’t know either population standard deviation – and the two samples are not paired. Following the Standard Error Decision Tree, we arrive in a box stating

that $S_E = \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$ and degrees of freedom, $df = \min\{n_1 - 1, n_2 - 1\}$. Remember that being in a box giving information about degrees of freedom tells us that we must use the t distribution rather than the standard normal distribution for computing our p -value.

Note that since we are testing whether $\mu_{\text{men}} < \mu_{\text{women}}$, we are thinking that $\mu_{\text{men}} - \mu_{\text{women}}$ would give us a negative value. This means we have a one-tailed test, with the critical region (rejection region) in the lower (left) tail. We notice now that we have a one-tailed test, with $df = \min\{9 - 1, 14 - 1\} = 8$, and $\alpha = 0.05$. Now we must compute a p -value for the test. Since the population standard deviations of ages are unknown, we must use the t -distribution (that is, we must use the T.DIST() function) to find the p -value. In order to use T.DIST() we must first convert our values into a test statistic.

We compute the test statistic below:

$$\begin{aligned}
 t &= \frac{(\text{point estimate}) - (\text{null value})}{S_E} \\
 &\quad \text{Difference in Sample Averages} \\
 t &= \frac{\overbrace{(39 - 44)} - 0}{\sqrt{\frac{(4.5)^2}{14} + \frac{(6.2)^2}{9}}} \\
 &\approx \frac{-5}{2.39} \\
 &= -2.09
 \end{aligned}$$



Since we now have the test statistic, we will use `T.DIST(boundary, df, TRUE)` in order to compute the p -value. See the image to the right.

$$\begin{aligned}
 p.value &= T.DIST(-2.09, 8, TRUE) \\
 &\approx 0.035
 \end{aligned}$$

Step 4: Make a Conclusion

Since our p -value is less than α (0.05), it is unlikely that our sample was taken from distributions with equal mean ages. That is, we reject H_0 and accept H_a – we have significant evidence to suggest that the population mean age of men on Death Row is less than the population mean age of women.