Example 1 (Hypothesis Test) *A recent study looked at average ages of male and female inmates on Death Row. The random sample of 9 women and 14 men on death row resulted in the following:*

	Men	***Women***
$\bar{x}$	*39*	*44*
s	*4.5*	*6.2*

Conduct a hypothesis test to determine whether there is strong evidence to suggest that the men on Death Row are, on average, younger than women on Death Row. Use $\alpha = 0.05$.

Solution. Notice that we are asked to conduct a hypothesis test – keep in mind that, even if the prompt didn't request a hypothesis test explicitly, any request for "evidence" leads us to conduct a hypothesis test.

Step 1: Hypotheses:

$$H_0: \ \mu_{women} = \mu_{men}$$
$$H_a: \mu_{men} < \mu_{women}$$

Step 2: Set level of significance

Let $\alpha = 0.05$ means that unless observing a sample like ours (at least as favorable to H_a) is less than 5% likely, then we cannot reject the null hypothesis (H_0).

Step 3: Compute test statistic and p-value.

Notice that we are conducting a hypothesis test involving means (μ's) – we have two samples – we don't know either population standard deviation – and the two samples are not paired. Following the Standard Error Decision Tree, we arrive in a box stating that $S_E = \sqrt{\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2}}$ and degrees of freedom, $df = \min\{n_1 - 1, n_2 - 1\}$. Remember that being in a box giving information about degrees of freedom tells us that we must use the t distribution rather than the standard normal distribution for computing our p-value.

Note that since we are testing whether $\mu_{men} < \mu_{women}$, we are thinking that $\mu_{men} - \mu_{women}$ would give us a negative value. This means we have a one-tailed test, with the critical region (rejection region) in the lower (left) tail. We notice now that we have a one-tailed test, with $df = \min\{9-1, 14-1\} = 8$, and $\alpha = 0.05$. Now we must compute a p-value for the test. Since the population standard deviations of ages are unknown, we must use the t-distribution (that is, we must use the `pt` function) to find the p-value. Unfortunately, in order to use `pt` we must first convert our values into a test statistic.

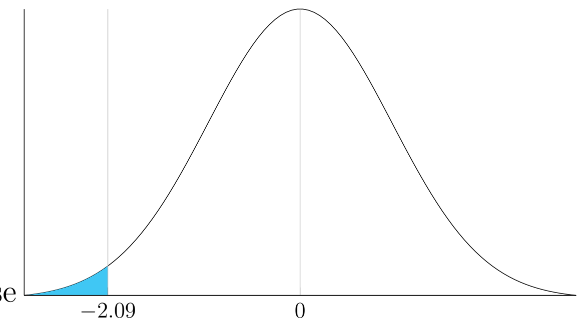
We compute the test statistic below:

$$t = \frac{(point\ estimate) - (null\ value)}{S_E}$$

$$t = \frac{\overbrace{(39 - 44)}^{\text{Difference in Sample Averages}} \quad -0}{\sqrt{\frac{(4.5)^2}{14} + \frac{(6.2)^2}{9}}}$$

$$\approx \frac{-5}{2.39}$$

$$= -2.09$$



Since we now have the test statistic, we will use `pt(boundary, df)` in order to compute the p-value. See the image to the right.

$$p.value = pt(-2.09, 8)$$
$$\approx 0.035$$

Step 4: <u>Make a Conclusion</u>

Since our p-value is less that α (0.05), it is unlikely that our sample was taken from distributions with equal mean ages. That is, we reject H_0 and accept H_a – we have significant evidence to suggest that the population mean age of men on Death Row is less than the population mean age of women.