

Example 1 (Confidence Interval) *A recent random sample of 30 customers who switched to a car insurance company boasting that new customers save an average of \$534 per year resulted in an average savings of \$525 with a standard deviation of \$40. Find a 90% confidence interval for the true mean savings of customers switching to this company.*

Solution. We are asked to compute a 90% confidence interval for the true average savings (μ). From the Standard Error Decision Tree handout we can recall the general formula for a confidence interval is

$$(\text{point estimate}) \pm (\text{critical value}) (\text{standard error})$$

We immediately have a point estimate for the true mean – it is a savings of \$525 which we found from our sample. We need to identify the “critical value” and the “standard error”. We use the distribution tables to identify the critical value and can identify how we should compute the standard error using the Standard Error Decision Tree.

- ◇ **Standard Error:** Notice that we have one sample and that σ is unknown. Using the Standard Error Decision Tree, this leads us to a box with $S_E = s/\sqrt{n}$ and $df = n - 1$. Recall that this means we are using the t -distribution.

$$S_E = \frac{40}{\sqrt{30}} \approx \$7.30$$

- ◇ **Critical Value:** Recall that all confidence intervals are two-tailed. Additionally, if we want 90% confidence, then $\alpha = 1 - 0.90 = 0.10$. We have $df = 30 - 1 = 29$. We can use the `T.INV()` function in our spreadsheet software to find the *critical value*. We need to provide `T.INV()` with the area to the left of the critical value we’re finding, along with the degrees of freedom for the distribution. Remember that a 90% confidence interval captures 90% of the area in the center of the distribution, leaving each tail with 5% of the area. This means that the total area to the left of the cutoff value for the upper tail is $0.90 + 0.05 = 0.95$. Using `T.INV(0.95, 29)` gives a critical value of approximately 1.699.

Since we’ve got all of the pieces: point estimate, critical value, and standard error, we build the confidence interval using the general form.

$$\begin{aligned} & (\text{point estimate}) \pm \underbrace{(\text{critical value}) (\text{standard error})}_{\text{Margin of Error}} \\ \implies & 525 \pm 1.7 (7.30) \\ \implies & (512.59, 537.41) \end{aligned}$$

Notice that this means “We are 90% confident that the true average savings that a customer switching to this company can expect is between \$512.59 and \$537.41”. This supports the company’s claim that new customers save \$534 when switching from their current insurance provider.