Example 1 (Confidence Interval) *A recent random sample of* 30 *customers who switched to a car insurance company boasting that new customers save an average of* \$534 *per year resulted in an average savings of* \$525 *with a standard deviation of* \$40*. Find a* 90% *confidence interval for the true mean savings of customers switching to this company.*

Solution. We are asked to compute a 90% confidence interval for the true average savings (μ). From the Standard Error Decision Tree handout we can recall the general formula for a confidence interval is

$$(\textit{point estimate}) \pm (\textit{critical value})\,(\textit{standard error})$$

We immediately have a point estimate for the true mean – it is a savings of \$525 which we found from our sample. We need to identify the "critical value" and the "standard error". We use the distribution tables to identify the critical value and can identify how we should compute the standard error using the Standard Error Decision Tree.

◇ **Standard Error:** Notice that we have one sample and that σ is unknown. Using the Standard Error Decision Tree, this leads us to a box with $S_E = s/\sqrt{n}$ and $df = n - 1$. Recall that this means we are using the t-distribution.

$$S_E = \frac{40}{\sqrt{30}} \approx \$7.30$$

◇ **Critical Value:** Recall that all confidence intervals are two-tailed. Additionally, if we want 90% confidence, then $\alpha = 1 - 0.90 = 0.10$. We have $df = 30 - 1 = 29$. From the t-table, we find a critical value of 1.70
If we want to do better than this, we can use `R`'s `qt` function. Remember that `qt`, like `qnorm`, gives cutoff values for percentiles. If we wish to have a better approximation for the `critical value` we use `qt(0.05, 29)` $\approx$ `1.699` we use 0.05 instead of the 0.10 because a confidence interval splits α equally among the two tails of the distribution. Similarly, we ignore that the result is negative because the confidence interval already takes this into account.

Since we've got all of the pieces: point estimate, critical value, and standard error, we build the confidence interval using the general form.

$$(\textit{point estimate}) \pm \underbrace{(\textit{critical value})\,(\textit{standard error})}_{\text{Margin of Error}}$$

$$\implies 525 \pm 1.7\,(7.30)$$

$$\implies (512.59, 537.41)$$

Notice that this means "We are 90% confident that the true average savings that a customer switching to this company can expect is between \$512.59 and \$537.41". This supports the company's claim that new customers save \$534 when switching from their current insurance provider.