Practice Problems Notebook

MAT241 Class

In this notebook, we'll work through several practice problems. The problems appear in no particular order, and each one is best addressed using a probability distribution/tool that we've encountered so far. As a reminder, we've seen

- Discrete, counting-based probability
- The binomial distribution in the case where
  - We repeat the same experiment n times (we run n trials)
  - Each trial has two possible outcomes (success or failure)
  - The probability of success does not change from one trial to the next (independence)
- The Normal distribution for continuous variables which are, at least, *approximately* normally distributed

We have the following useful functionality:

- If X is the number of successes observed in a *binomial experiment* with n trials and probability of success equal to p, then
  - $\mathbb{P}\overbrace{[X = k]}^{\text{Exactly } k \text{ successes}} = \texttt{dbinom(k, n, p)}$
  - $\mathbb{P}\overbrace{[X \leq k]}^{\text{At most } k \text{ successes}} = \texttt{pbinom(k, n, p)}$
  - The *expected number* of successes is $\mathbb{E}[X] = np$
  - The *standard deviation* in number of successes is $s_X = \sqrt{np(1-p)}$
- If X is approximately normally distributed with mean μ and standard deviation σ (that is, if $X \sim N(\mu,\ \sigma)$), then
  - $\mathbb{P}[X \leq k] = \texttt{pnorm(k, mean = }\mu\texttt{, sd = }\sigma\texttt{)}$
  - The cutoff value (quantile) q such that the probability of observing a value at most q is p (that is, $\mathbb{P}[X \leq q] = p$) is given by $\texttt{qnorm(p, mean =}\mu\texttt{, sd = }\sigma\texttt{)}$

> **Note**
>
> This document contains "answers" in the form of probabilities or quantiles so that you can check your work. Looking at the answers first may give away a partial strategy for solving the problem, so be sure that you only look at the answers to validate your work.

Example Scenarios

Scenario 1: Smartphone Battery Life

The lifespan of a certain smartphone battery is approximately normally distributed with a mean of 18 months and a standard deviation of 3 months.

i) What is the probability that a battery has a lifespan exceeding 2 years (24 months)?
ii) What is the probability that a battery has a lifespan between 16 months and 2 years?
iii) The manufacturer wants to put a warranty on their batteries, but they want to replace no more than 3% of batteries via warranty. What is the cutoff for the lifespan of these shortest lasting batteries?

Scenario 2: Manufacturing Defects

A factory has a defect rate of 3% in the products it manufactures. A Quality Control Engineer indicates that defects seem to happen at random and are independent of one another. Inspectors randomly select 100 products.

i) What is the probability that exactly 5 products have defects?
ii) What is the probability that at most 5 products have defects?
iii) What is the probability that at least 5 products have defects?
iv) What is the probability that more than 5 but at most 10 products have defects?
v) What is the *expected number* of products with defects out of the number inspected?
vi) What is the *standard deviation* in number of products with defects when inspecting 100 products?
vii) Should the inspectors be surprised if they observe 12 products with defects out of the 100 inspected?

Scenario 3: Coffee Shop Customers

A new coffee shop estimates that each customer has a 65% chance of buying a pastry along with their drink. On a given morning, 40 customers visit the shop.

i) What is the probability that exactly 25 of them buy a pastry?
ii) What is the probability that fewer than 20 of them buy a pastry?
iii) What is the probability that at least 30 of them buy a pastry?
iv) What is the probability that at least 20 but no more than 30 buy a pastry?

Scenario 4: Streaming Session Length

The length of time (in minutes) that a user spends on a streaming platform per session is approximately normally distributed with a mean of 42 minutes and a standard deviation of 12 minutes.

i) What is the probability that a user spends less than 30 minutes streaming?
ii) What is the probability that a user spends more than 60 minutes streaming?
iii) What is the probability that a user spends between 35 and 50 minutes streaming?
iv) What is the cutoff for the top 10% of longest streaming sessions?

Scenario 5: Esports Reaction Times

Reaction times for professional esports players (in milliseconds) are approximately normally distributed with a mean of 250 ms and a standard deviation of 40 ms.

i) What proportion of players have reaction times faster than 200 ms?
ii) What proportion of players have reaction times slower than 300 ms?
iii) What proportion of players have reaction times between 230 ms and 280 ms?
iv) What is the cutoff for the fastest 5% of players?

Scenario 6: Voter Participation

In a survey, you ask 200 randomly selected people whether they plan to vote in an upcoming election. Historically, 55% of people vote.

i) What is the probability that fewer than 85 people are planning on voting?
ii) What is the probability that at least 125 people are planning on voting?
iii) What is the probability that more than 85 but fewer than 125 people are planning on voting?

Scenario 7: Apple Picking

The weight of apples grown in an orchard is approximately normally distributed with a mean of 150 grams and a standard deviation of 20 grams.

i) What is the probability that a randomly selected apple weighs less than 100 grams?
ii) What is the probability that a randomly selected apple weighs more that 175 grams?
iii) What is the probability that a randomly selected apple weighs between 160 and 195 grams?
iv) What is the cutoff for the lightest 5% of apples?
v) What is the cutoff for the heaviest 1% of apples?

Scenario 8: Gamer Loot Drops

In a certain video game, a rare loot drop occurs with a probability of 8% each time a chest is opened. A player opens 50 chests in a weekend.

i) What is the probability that they receive exactly 5 rare drops?
ii) What is the probability that they receive at least 8 rare drops?
iii) What is the probability that they receive no more than 3 rare drops?
iv) What is the *expected number* of rare drops?
v) What is the *standard deviation* in number of loot drops?

Scenario 9: Trivia Night Team Scores

Team scores at a weekly trivia night follow an approximately normal distribution with a mean of 72 points and a standard deviation of 10 points.

i) What proportion of teams score above 80 points?
ii) What proportion of teams score below 60 points?
iii) What proportion of teams score between 65 and 85 points?
iv) What is the cutoff score for the top 15% of teams?

Scenario 10: Daily Steps

A student tracks their daily step count over a semester and finds it is approximately normally distributed with a mean of 8,200 steps and a standard deviation of 1,600 steps.

i) What is the probability that they walk fewer than 6,000 steps on a randomly selected day?
ii) What is the probability that they walk more than 10,000 steps?
iii) What is the probability that they walk between 7,500 and 9,000 steps?
iv) What step count represents the 90th percentile?

Scenario 11: Spotify Playlist Length

The total length (in minutes) of user-created playlists on a music app is approximately normally distributed with a mean of 240 minutes and a standard deviation of 60 minutes.

i) What is the probability that a playlist is shorter than 180 minutes?
ii) What is the probability that a playlist is longer than 300 minutes?
iii) What is the probability that a playlist is between 200 and 260 minutes?
iv) What playlist length represents the top 20% longest playlists?

Scenario 12: Basketball Free Throws

A basketball player makes free throws with a probability of 78%. In a game, they take 12 free throws.

i) What is the probability that they make exactly 10 free throws?
ii) What is the probability that they make all of their free throws?
iii) What is the probability that they make at least 8 free throws?
iv) What is the *expected number* of free throws they make?
v) What is the *standard deviation* in the number of free throws they make?
vi) Should we be surprised if they make only 7 free throws?

Answers

Answers are provided below. All probabilities are rounded to four decimal places. Any other values are rounded to two decimal places, if necessary.

Scenario 1: Smartphone Battery Life

i) 0.0228
ii) 0.7248
iii) 12.36 months

Scenario 2: Manufacturing Defects

i) 0.1013
ii) 0.9192
iii) 0.1821
iv) 0.0806
v) 3
vi) 1.71
vii) 12 defects is unlikely because it is so many standard deviations above the expected outcome (we generally use two standard deviations as the cutoff)

Scenario 3: Coffee Shop Customers

i) 0.1226
ii) 0.0173
iii) 0.1215
iv) 0.9183

Scenario 4: Streaming Session Length

i) 0.1587
ii) 0.0668
iii) 0.4677
iv) 57.38 minutes

Scenario 5: Esports Reaction Times

i) 0.1056
ii) 0.1056
iii) 0.4648
iv) 184.21 ms

Scenario 6: Voter Participation

i) 0.0002
ii) 0.0191
iii) 0.9806

Scenario 7: Apple Picking

i) 0.0062
ii) 0.1056

iii) 0.2963
iv) 117.10 grams
v) 196.53 grams

Scenario 8: Gamer Loot Drops

i) 0.1629
ii) 0.0438
iii) 0.4253
iv) 4 items
v) 1.92 items

Scenario 9: Trivia Night Team Scores

i) 0.2119
ii) 0.1151
iii) 0.6612
iv) 82.36 points

Scenario 10: Daily Steps

i) 0.0846
ii) 0.1303
iii) 0.3606
iv) 10250.48 steps

Scenario 11: Spotify Playslist Length

i) 0.1587
ii) 0.1587
iii) 0.3781
iv) 290.50 minutes

Scenario 12: Basketball Free Throws

i) 0.2663
ii) 0.0507
iii) 0.8979
iv) 9.36 baskets
v) 1.43 baskets
vi) No, 7 made is within two standard deviations of the expected value